ECON 217: Section Notes

Week 2

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January 13-15, 2021

Properties of E(•)

$$E[c] = c \text{ for any constant c}$$

$$E\left[cX\right] = cE\left[X\right]$$

$$E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E\left[X_i\right]$$

$$E[g\left(X\right)] = \begin{cases} \sum_{j=1}^{k} g\left(x_j\right) f\left(x_j\right) & \text{if } X \text{ is discrete} \\ \int_{-\infty}^{\infty} g\left(x\right) f\left(x\right) dx & \text{if } X \text{ is continuous} \end{cases}$$

Properties of Var(•) and Cov(•,•)

$$Var\left[cX\right] = c^2 Var\left[X\right]$$

$$StdDev\left[cX\right] = |c|\, StdDev\left[X\right]$$

$$Cov\left(X, c\right) = 0$$

$$Cov\left(cX, Y\right) = cCov\left(X, Y\right)$$

$$Cov\left(X, X\right) = Var[X]$$

$$Cov\left(aX + bY, cZ\right) = acCov\left(X, Z\right) + bcCov\left(Y, Z\right)$$

$$Var\left[X + Y\right] = Var\left[X\right] + Var\left[Y\right] + 2Cov\left(X, Y\right)$$

For independent random variables $X_1, ..., X_n$ and constants $a_1 ..., a_n$

$$Var\left[\sum_{i=1}^{n} a_i X_i\right] = \sum_{i=1}^{n} a_i^2 Var\left[X_i\right]$$

Examples

$$E(5X + X^2 - 7Y + 8) = 5E(X) + E(X^2) - 7E(Y) + 8$$

$$\begin{aligned} Var(5X + X^2 - 7Y + 8) &= Var(5X) + Var(X^2) + Var(-7Y) \\ &\quad + 2Cov(5X, X^2) + 2Cov(X^2, -7Y) + 2Cov(5X, -7Y) \\ &= 5^2 Var(X) + Var(X^2) + 7^2 Var(Y) \\ &\quad + 10Cov(X, X^2) - 14Cov(X^2, Y) - 70Cov(X, Y) \end{aligned}$$

Some handy examples

$$\begin{aligned} Cov(X, Y) &= E[(X - E[X])(Y - E[Y])] \\ &= E(XY) - E(X)E(Y) \\ Var(X) &= E[(X - E(X))^2] \\ &= E(X^2) - E(X)^2 \end{aligned}$$

Numerical questions

Consider the following joint distribution:

f(x,y)	Y=0	Y=1	Y=2
X=5	0.14	0.07	0.09
X=10	0.02	0.22	0.07
X=12	0.11	0.23	0.05

You can calculate expectations, variances, etc. by hand, but also more easily (less prone to mistakes) in a **spreadsheet**.