## Problem Set 3

## Econ 211C

$$Y_t = 1.3Y_{t-1} - 0.4Y_{t-2} + \varepsilon_t + 0.7\varepsilon_{t-1} + 0.1\varepsilon_{t-3} - 0.5\varepsilon_{t-4} - 0.2\varepsilon_{t-5}$$

where  $\varepsilon_t \sim WN(0,1)$ .

- (a) (5 points) What are the exact, finite-sample, one-step forecast coefficients?
- (b) (8 points) Simulate N=105 observations for this process. Starting with  $Y_{100}$ , compute and report one-step forecasts for  $Y_{101}, \ldots, Y_{105}$ . When computing the forecasts for  $t \geq 102$ , use your previously computed forecasts as data, rather than the actual values that you originally simulated.
- (c) (8 points) Repeat part (b) 1000 times. What is the mean squared error of your forecast for  $Y_{105}$ ?
- (d) (5 points) What are the exact, finite-sample, five-step forecast coefficients?
- (e) (6 points) Compute and report a five-step forecast for  $Y_{105}$ .
- (f) (6 points) Repeat part (e) 1000 times. What is the mean squared error of your forecast for  $Y_{105}$ ?

For the remainder of the problem, assume that you do not know the true coefficients of the process, but that you do know that it is an ARMA(2,5).

- (g) (5 points) Use the first 100 observations,  $\{y_t\}_{t=1}^{100}$  to estimate the ARMA(2,5). What are the parameter estimates?
- (h) (5 points) Repeat part (a), using your estimates.
- (i) (5 points) Repeat parts (b) and (c), using the same estimates (without updating) for each one-step forecast.
- (j) (7 points) Repeat parts (b) and (c), updating your estimates with each forecast. That is, compute the forecast for  $Y_{101}$ , using estimates obtained from  $\{y_t\}_{t=1}^{100}$ , compute the forecast for  $Y_{102}$  using estimates obtained from  $\{y_t\}_{t=2}^{100}$  and your forecast  $\hat{Y}_{101}$ , etc.
- (k) (5 points) Repeat part (d), using your estimates.
- (l) (5 points) Repeat parts (e) and (f), using the your estimates.

The file ps3Dat.csv contains data on 1 minute returns and order flow for the EUR/USD exchange rate on 13 Nov 2013. The first column of the dataset contains a date/time stamp, the second column reports returns over each minute between 9:30 am and 4:00 pm EST, and the last column reports order flow for the same minutes. Order flow can be thought of as signed volume – trades occurring at the lowest offer prices are counted as a positive number of traded contracts during the time interval, and trades occurring at the highest bid prices are counted as a negative number of traded contracts. Volume, on the other hand, counts all traded contracts positively, regardless of which side of the order book the transactions take place.

- (a) (10 points) Estimate a VAR(2) model for EUR/USD returns and order flow. Write the equations of the full model, substituting estimated values for the parameters. (Hint: the estimation can be done equation by equation).
- (b) (10 points) Rewrite the VAR(2) as a VAR(1), again substituting estimates for parameters.
- (c) (10 points) What is the matrix that will orthogonalize the error vector of the VAR(2)?