Problem Set 2

Econ 211C

Question 1 .... *35 points*

Suppose $\{Y_i\}_{i=1}^n \overset{i.i.d.}{\sim} Weibull(k, \lambda)$:

$$f_{Y_i}(y_i|k,\lambda) = \frac{k}{\lambda}\left(\frac{y_i}{\lambda}\right)^{k-1} \exp\left\{-\left(\frac{y_i}{\lambda}\right)^k\right\}.$$

(a) (10 points) Derive the log likelihood, $\ell(k, \lambda|\boldsymbol{y})$.

(b) (10 points) Derive the maximum likelihood estimators, $\hat{k}$ and $\hat{\lambda}$.

(c) (15 points) Derive the information matrix. What is the observed information matrix? Given estimates, $\hat{k}$ and $\hat{\lambda}$, what would approximations of the variances of the estimates be?

Question 2 .... *35 points*

Consider an $AR(2)$ process

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-1} + \varepsilon_t$$

where $\varepsilon_t \overset{i.i.d}{\sim} \mathcal{N}(0,1)$ and where $\boldsymbol{\phi} = (\phi_1, \phi_2)' = (1.3, -0.41)'$.

(a) (25 points) Simulate 30 observations from this process and compute the least-squares estimates for three regressions:

$$Y_t = \phi_1 Y_{t-1} + \varepsilon_t$$
$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$$
$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \varepsilon_t.$$

Repeat the simulation/estimation 1000 times and report the means and standard deviations of each set of estimates in a table. Include your `R` code with your solution.

(b) (5 points) Repeat part (a), simulating $N = 1000$ observations instead of $N = 30$ observations at each iteration. Report the estimates in the same table as part (a).

(c) (5 points) Repeat part (b) with $N = 100,000$.

Question 3 .... *30 points*

Download daily adjusted closing prices for ticker XIV from finance.yahoo.com for dates 25 April 2014 to 24 April 2015. Find the best fitting $ARMA$ model for these data. Report the parameter estimates and standard errors and provide some interpretation.