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ECON 205C: Problem Set 4

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Problem 1

(a)

Suppose $\beta = \frac{1}{1+\rho}$. The value of being employed and exerting effort is as follows:

$$V_E = w(t) - \bar{e} + \beta[\delta V_U + (1-\delta)V_E] \tag{1}$$

$$\begin{aligned}(1+\rho)V_E &= (1+\rho)[w(t)-\bar{e}] + \delta V_U + (1-\delta)V_E \\ (\rho+\delta)V_E &= (1+\rho)[w(t)-\bar{e}] + \delta V_U\end{aligned}$$

$$\therefore \quad V_E = \frac{(1+\rho)[w(t)-\bar{e}] + \delta V_U}{\rho+\delta} \tag{2}$$

The value of being employed and shirking is as follows:

$$V_S = w(t) + \beta[(\delta+q)V_U + (1-\delta-q)V_S] \tag{3}$$

$$\begin{aligned}(1+\rho)V_S &= (1+\rho)w(t) + (\delta+q)V_U + (1-\delta-q)V_S \\ (\rho+\delta+q)V_S &= (1+\rho)w(t) + (\delta+q)V_U\end{aligned}$$

$$\therefore \quad V_S = \frac{(1+\rho)w(t) + (\delta+q)V_U}{\rho+\delta+q} \tag{4}$$

The value of being unemployed is as follows:

$$V_U = b(t) + \beta[aV_E + (1-a)V_U] \tag{5}$$

$$(1+\rho)V_U = (1+\rho)b(t) + aV_E + (1-a)V_U$$

$$\therefore \quad \rho V_U = (1+\rho)b(t) + a(V_E - V_U) \tag{6}$$

(b)

To look at the difference between the values of being employed and unemployed, we can find the difference between equations (1) and (5):

$$\begin{aligned}
V_E - V_U &= \big[w(t) - \bar{e} + \beta[\delta V_U + (1-\delta)V_E]\big] - \big[b(t) + \beta[aV_E + (1-a)V_U]\big] \\
&= w(t) - \bar{e} - b(t) + \beta[(\delta + a - 1)V_U + (1 - a - \delta)V_E] \\
(1+\rho)(V_E - V_U) &= (1+\rho)[w(t) - \bar{e} - b(t)] + (1 - a - \delta)(V_E - V_U) \\
(\rho + \delta + a)(V_E - V_U) &= (1+\rho)[w(t) - \bar{e} - b(t)] \\
\therefore \quad V_E - V_U &= \frac{(1+\rho)[w(t) - \bar{e} - b(t)]}{\rho + \delta + a}
\end{aligned}$$

The above equation shows us the difference between the values of being employed and unemployed does depend on $b(t)$. explanation ?

(c)

The no shirking condition assumes $V_E = V_S$. If we set equations (1) and (2) equal, we will see that:

$$\begin{aligned}
w(t) - \bar{e} + \beta[\delta V_U + (1-\delta)V_E] &= w(t) + \beta[(\delta + q)V_U + (1 - \delta - q)V_E] \\
-(1+\rho)\bar{e} + \delta V_U + (1-\delta)V_E &= (\delta + q)V_U + (1 - \delta - q)V_E \\
-(1+\rho)\bar{e} &= qV_U - qV_E \\
\therefore \quad V_E - V_U &= \frac{1+\rho}{q}\bar{e}
\end{aligned}$$

Our no shirking condition is $V_E - V_U = V_S - V_U = \frac{1+\rho}{q}\bar{e} > 0$. Now we can derive the equilibrium wage using equation (2) and equation (6).

$$\begin{aligned}
(\rho + \delta)V_E &= (1+\rho)[w(t) - \bar{e}] + \delta V_U \\
(1+\rho)w &= (1+\rho)\bar{e} + (\rho + \delta)V_E - \delta V_U \\
(1+\rho)w &= (1+\rho)\bar{e} + \rho V_U + (\rho + \delta)(V_E - V_U) \\
(1+\rho)w &= (1+\rho)\bar{e} + (1+\rho)b(t) + a(V_E - V_U) + (\rho + \delta)(V_E - V_U) \\
(1+\rho)w &= (1+\rho)[\bar{e} + b(t)] + (a + \rho + \delta)(V_E - V_U)
\end{aligned}$$

$$\therefore \quad w = b(t) + \frac{a + \rho + \delta + q}{q}\bar{e} \tag{7}$$

We can see that the equilibrium wage does depend on non-labor income $b(t)$.

(d)

The per period flow into unemployment is δNL and the per period flow out of unemployment is $a(\bar{L} - NL)$. In equilibrium these flows must be equal, so $a = \frac{\delta NL}{\bar{L} - NL}$.

(e)

Suppose $N(t) = L(t) + S(t)$ where $L(t)$ is the number of employees exerting effort and $S(t)$ is the number shirking.

$$\max A[e(t)N(t)]^{\alpha} - w(t)N(t) = \max A[\bar{e}L(t)]^{\alpha} - w(t)N(t)$$

Take the derivative respect to $N(t)$:

$$\alpha A[\bar{e}L(t)]^{\alpha-1}\bar{e} - w(t) = 0$$

Substitute equation (7) into the equation above:

$$\begin{aligned} \alpha A[\bar{e}L(t)]^{\alpha-1}\bar{e} - b(t) + \frac{a+\rho+\delta+q}{q}\bar{e} &= 0 \\ \alpha A L(t)^{\alpha-1}\bar{e}^{\alpha} - b(t) + \frac{a+\rho+\delta+q}{q}\bar{e} &= 0 \\ \therefore \quad L(t)^{\alpha-1} &= \frac{1}{\alpha A}\left[b(t)\bar{e}^{-\alpha} - \frac{a+\rho+\delta+q}{q}\bar{e}^{1-\alpha}\right] \end{aligned}$$

for given a.

$-\frac{1}{2}$

As $0 < \alpha \leq 1$, we can see that the equilibrium employment decreases when the non-labor income, $b(t)$, increases. Becuase $b(t)$ represents the opportunity cost of being employed, and increase in $b(t)$ would force employers to increase wages which would in turn lead to a lower equilibrium employment level.

Problem 2

(a)

From the new Keynesian model with sticky prices and wages, the equation for price inflation can be solved forward as

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \kappa\, \widehat{mc}_t \\ &= E_t\Big[\beta^2 \pi_{t+2} + \kappa\beta\, \widehat{mc}_{t+1} + \kappa\, \widehat{mc}_t\Big] \\ &= E_t\Big[\beta^3 \pi_{t+3} + \kappa\, \widehat{mc}_{t+2} + \kappa\beta\, \widehat{mc}_{t+1} + \kappa\, \widehat{mc}_t\Big] \\ &\ \vdots \\ &= \kappa E_t \sum_{i=0}^{\infty} \beta^i\, \widehat{mc}_{t+i}, \end{aligned}$$

where $\widehat{mc}_t$ denotes the real marginal cost in terms of deviation from the flexible-price equilibrium. Note that the real marginal cost can also be expressed as $\widehat{mc}_t = \widehat{\omega}_t - \widehat{mpl}_t$. So we can see that price inflation is driven by $(\widehat{\omega}_t - \widehat{mpl}_t)$. where ω_t and mpl_t denote real wage and marginal product of labor, respectively.

Now, the wage inflation can be solved forward as

$$\begin{aligned}
\pi_t^w &= \beta E_t \pi_{t+1}^w + \kappa_w(\widehat{mrs}_t - \widehat{\omega}_t) \\
&= E_t\Big[\beta^2 \pi_{t+2}^w + \kappa_w \beta(\widehat{mrs}_{t+1} - \widehat{\omega}_{t+1}) + \kappa_w(\widehat{mrs}_t - \widehat{\omega}_t)\Big] \\
&= E_t\Big[\beta^3 \pi_{t+3}^w + \kappa_w(\widehat{mrs}_{t+2} - \widehat{\omega}_{t+2}) + \kappa_w \beta(\widehat{mrs}_{t+1} - \widehat{\omega}_{t+1}) + \kappa_w(\widehat{mrs}_t - \widehat{\omega}_t)\Big] \\
&\ \ \vdots \\
&= \kappa_w E_t \sum_{i=0}^{\infty} \beta^i (\widehat{mrs}_{t+i} - \widehat{\omega}_{t+i}),
\end{aligned}$$

where $\widehat{mrs}_t$ and $\widehat{\omega}_t$ denote marginal rate of substitution (between consumption and leisure) and real wage, respectively, in terms of deviation from flexible-price equilibrium. So we can see that wage inflation is driven by $(\widehat{mrs}_t - \widehat{\omega}_t)$.

(b)

The factors generating inefficiencies in this economy are:

(1) Imperfect competition in the goods market and the labor market generates inefficiency in the steady state by causing wedges between mpl_t and w_t and between w_t and mrs_t, respectively.

(2) Dispersion of relative prices (wages) causes inefficiency in households' utility (firms' profit) as disutility from consuming more expensive goods (labor) is greater than the utility from consuming less expensive goods (labor).

(c)

When there is rigidity in only one of the prices and wages, monetary policy can eliminate the inefficiency from price dispersion as real wage adjusts to ensure labor market equilibrium in respond to shocks. However, when both wages and prices are sticky, monetary policy cannot keep the output gap at zero when facing with productivity shocks. Even if the monetary policy stabilizes the price level, the sticky wages prevent the real wage from adjusting to ensure labor market equilibrium. Similarly, even when the wage level is stabilized, the real wages cannot move to adjust to the equilibrium. Even though the output gap falls, the output doesn't. It means that the output doesn't rise as much as it does in flexible-wage-sticky-price. So, the effect of a cut in the interest rate central bank is not enough for the real wage to rise to the full extent.

(d)

λ_w has an inverse relationship with $\frac{\kappa_w}{\kappa_p}$. If wages are more stickier than prices (i.e. smaller κ_w), then the policy's loss function should have more weight on wage inflation. $\frac{\lambda_w}{\lambda_\pi} = \frac{\frac{\kappa_p}{\theta_p}}{\frac{\kappa_w}{\kappa_w}}$. The weight also depends on the elasticity of the goods and the labor market.

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How? explain.

(e)

?

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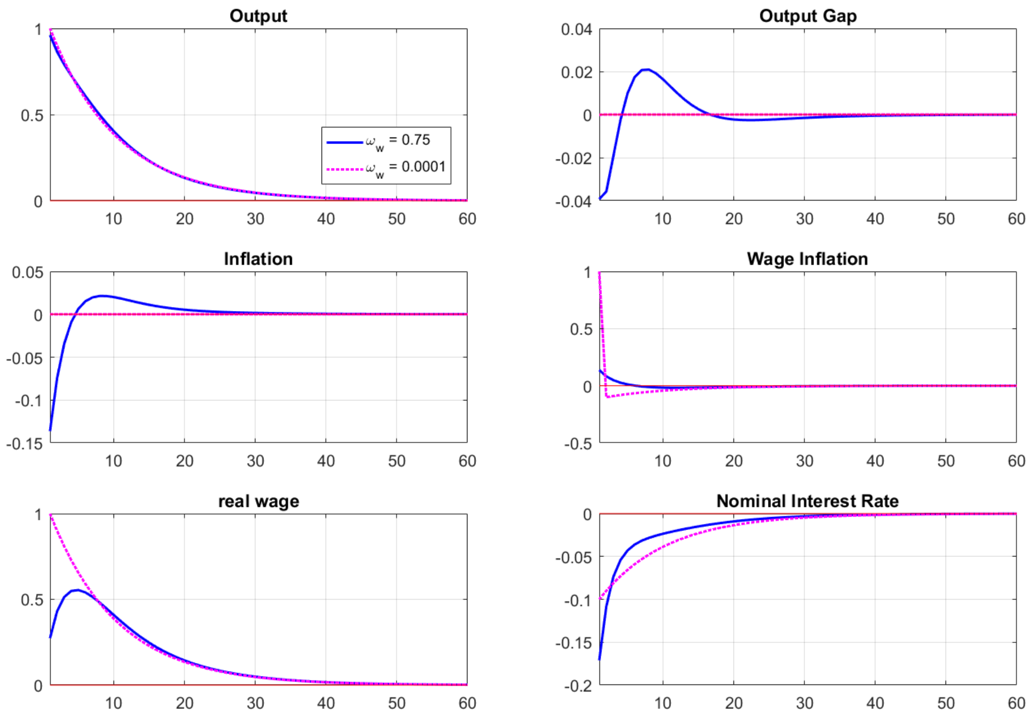

Your code is correct but it does not generate these results for $\omega = 0.0001$. Output response should be 0.

Figure 1: Responses to a positive productivity shock

Problem 3

(a)

If free entry in vacancy posting is assumed, then in equilibrium companies will post until $\beta q V^J = c$, which would mean $V^V = 0$.

(b)

Since we found that $V^V = 0$ in equilibrium assuming free entry, we can rearrange the first equation to get $V^J = \frac{c}{\beta q}$. As it becomes easier to fill a vacant job position, the marginal value of a filled job decreases.

(c)

If we think of the joint surplus to the worker and the firm of being in a match as being $V^j + V^e$, then

$$\begin{aligned} V^J + V^e &= \mu x - w + \beta(1-s)V^J + w - b + \beta(1-s)V^e \\ (1-\beta(1-s))[V^J + V^e] &= \mu x - b \\ V^J + V^e &= \frac{\mu x - b}{1-\beta(1-s)} \end{aligned}$$

Because the wage is being transferred from the firm to the worker without any being lost, the joint surplus of the firm and the worker is unaffected by the wage.

(d)

We will begin by maximizing $(V^J)^{1-\eta}(V^e)^\eta$, which can be written as:

$$max \left(\frac{\mu x - w}{1-\beta(1-s)}\right)^{1-\eta}\left(\frac{w-b}{1-\beta(1-s)}\right)^\eta$$

The first order condition with respect to w is:

$$\frac{\eta-1}{1-\beta(1-s)}\left(\frac{\mu x - w}{1-\beta(1-s)}\right)^{-\eta}\left(\frac{w-b}{1-\beta(1-s)}\right)^\eta + \frac{\eta}{1-\beta(1-s)}\left(\frac{\mu x - w}{1-\beta(1-s)}\right)^{1-\eta}\left(\frac{w-b}{1-\beta(1-s)}\right)^{\eta-1} = 0$$

$$\begin{aligned} \frac{\eta}{1-\beta(1-s)}\left(\frac{\mu x - w}{1-\beta(1-s)}\right)^{1-\eta}\left(\frac{w-b}{1-\beta(1-s)}\right)^{\eta-1} &= \frac{1-\eta}{1-\beta(1-s)}\left(\frac{\mu x - w}{1-\beta(1-s)}\right)^{-\eta}\left(\frac{w-b}{1-\beta(1-s)}\right)^\eta \\ \eta\left(\frac{\mu x - w}{1-\beta(1-s)}\right) &= (1-\eta)\left(\frac{w-b}{1-\beta(1-s)}\right) \\ \eta(\mu x - w) &= (1-\eta)(w-b) \end{aligned}$$

We can thus see that we have arrived at $w = (1-\eta)b - \eta\mu x$.