8/10

ECON 205C: Problem Set 3

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Problem 1

(a)

A positive productivity shock increases the flexible price output level. However, the domestic inflation rule doesn't respond directly to the change in the output gap. The negative output gap pushes down domestic product price inflation.

With $\sigma = 1$, the flexible output equation in open economy resembles closely to the closed economy. Flexible output increases due to a productivity shock which leads to a fall in the domestic prices P_{ht}.

Real wage of domestic workers increase. However, $\sigma = 1$ condition ensures that substitution and wealth effect cancel out and labor supply doesn't change. To maintain same labor input, nominal wages go down. Domestic firms reduce their prices as there is a decrease in their labor cost.

The lower domestic product price induces a higher terms of trade, which is the ratio of the price of foreign-produced consumption goods to home-produced consumption. As the terms of trade gets higher the foreign-produced consumption goods becomes more expensive in home country. Initially, a rise in foreign product price raises CPI inflation. A rise in CPI inflation increases the exchange rate to compensate for the effect in falling prices (Law of one price holds).

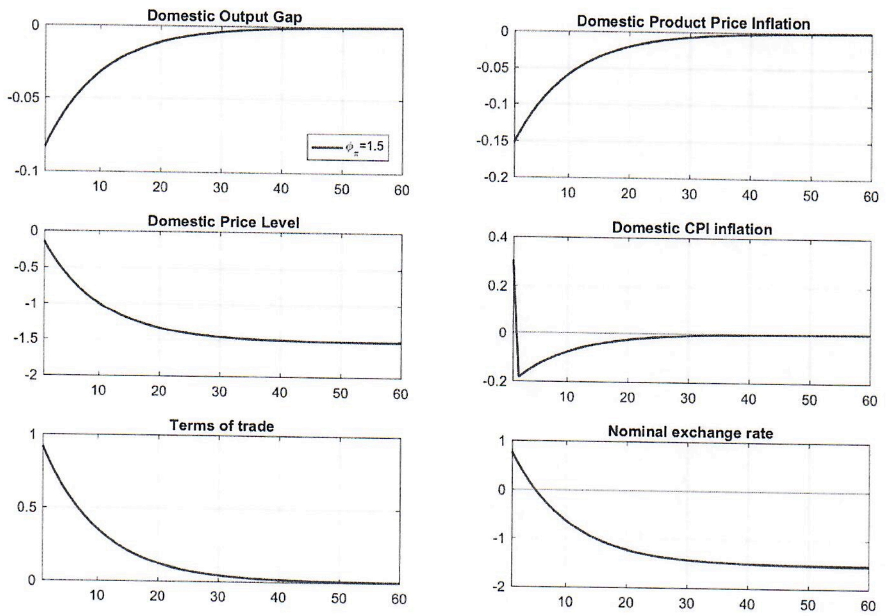

Figure 1: The responses with the policy rule policy $i_t = 1.5\pi_{h,t}$

(b)

As the policy responses stronger to domestic product price inflation, it stablizes both domestic product price inflation and output gap. The stronger response (higher ϕ_π) to domestic product price inflation lowers the value of the loss. The value of the loss function for $\phi_\pi = 1.5$ is 0.1232 and the value of the loss function for $\phi_\pi = 5$ is 0.0031.

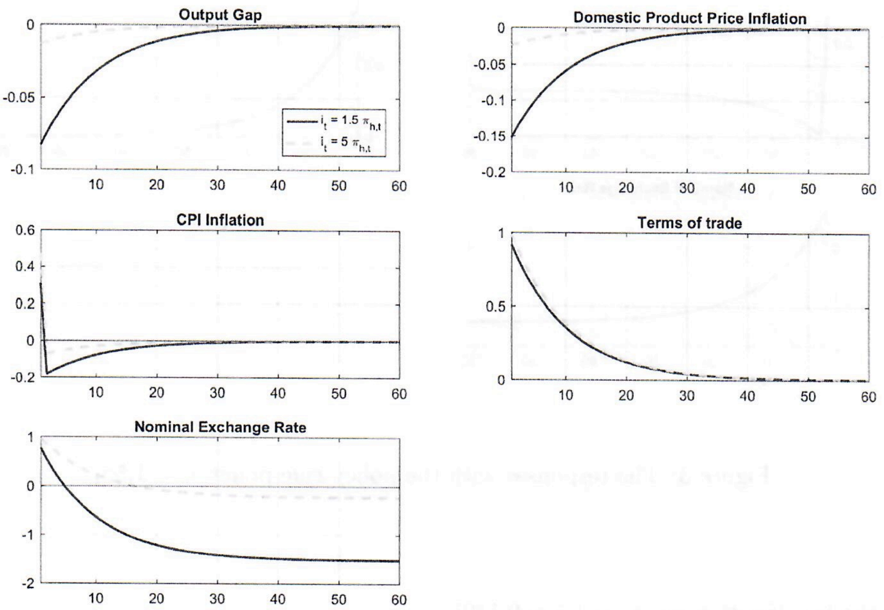

Figure 2: The responses with the policy rule policy $i_t = 5\pi_{h,t}$

(c)

Assuming the policy is given by $i_t = 1.5\pi_t$, the value of the loss function is 0.0608.

The nominal interest rate reacts to the CPI inflation. Targeting the CPI inflation stablizes the CPI inflation. The CPI inflation can be obtained from $\pi_t = \pi_{h,t} + \gamma(s_t - s_{t-1})$ and it includes domestic product price inflation and the difference of terms of trade. To stabilize the CPI inflation both the domestic price and the difference of terms of trade need to be controlled.

The graph of the output gap shows that the increase in output is restricted initially that does not increase the terms of trade too high. And then, the output level increase as much as the initial increase in the flexible price output level. This induce a higher terms of trade after the initial period.

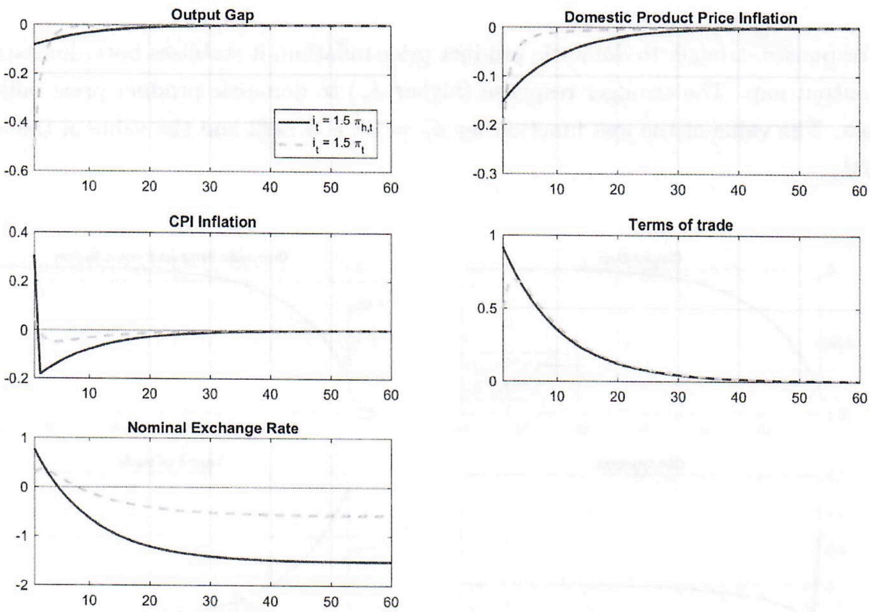

Figure 3: The responses with the policy rule policy $i_t = 1.5\pi_t$

(d)

The value of the loss function for $\phi_\pi = 1.5$ is 0.1405.

The loss is higher in this case as compared to (a) and (c). A reduction in domestic product price inflation (in (a)) called for an nominal interest rate cut. However, the exchange rate increases due to increase in CPI. If nominal interest rate is pegged to nominal exchange rate along with domestic product price inflation, monetary policy would require the rate cut to be smaller than earlier. The stabilising effect of the interest rate on domestic product price inflation and output gap would be smaller. So, loss is higher.

Table 1: The value of the loss function

Policy	L	σ_x^2	$\sigma_{\pi_h}^2$
$i_t = 1.5\pi_{h,t}$	0.1232	0.0365	0.1229
$i_t = 1.5\pi_t$	0.0608	0.3231	0.0581
$i_t = 1.5\pi_{h,t} + 10e_t$	0.1405	0.7203	0.1345

As the loss function is given by $\sigma_{\pi_h}^2 + \lambda\sigma_x^2$ where $\lambda = 0.0083$, the lowest value of the loss function can be acheived by the policy $i_t = 1.5\pi_t$. The variance of the output gap is the smallest when the policy is set as $i_t = 1.5\pi_{h,t}$. The variance of the domestic product price inflation is the smallest when the policy is set as

$i_t = 1.5\pi_t$. As the loss function assigns huge weight on the variance of the domestic product price inflation, the policy $i_t = 1.5\pi_t$ acheives the lowest value of the loss function.

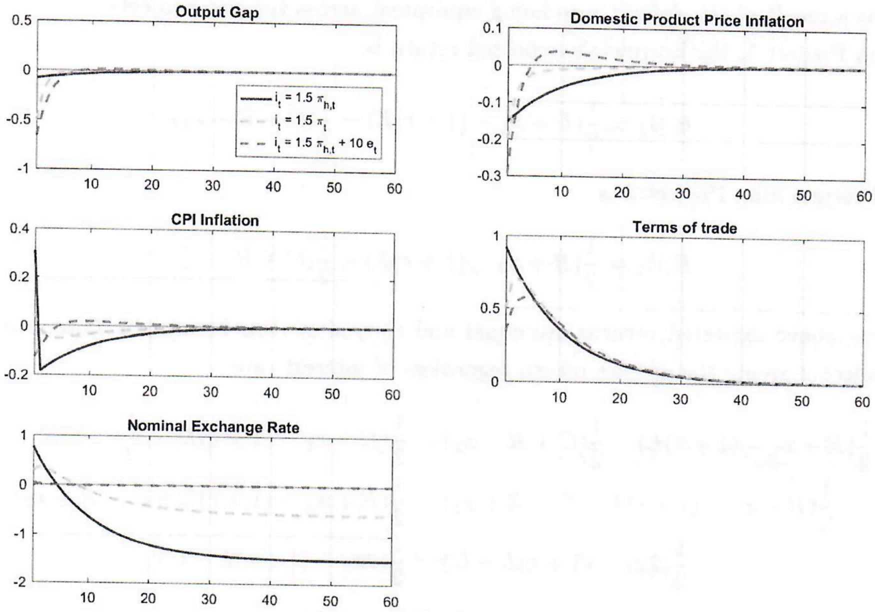

Figure 4: The responses with the policy rule $i_t = 1.5\pi_{h,t} + 10e_t$

Problem 2

(a)

For Project i (i = 1,2), the lender's expected return is $\frac{1}{2}(1+r)L + \frac{1}{2}(C + R - x_i)$, which must be equal to r. By solving for r,

$$\begin{aligned}
\frac{1}{2}(1+r)L + \frac{1}{2}(C+R-x_i) &= r \\
\frac{1}{2}(C+R-x_i) &= r - \frac{1}{2}(1+r)L \\
\frac{1}{2}(C+R-x_i+L) &= (1-\frac{1}{2}L)r \\
r &= \frac{C+R-x_i+L}{2((1-\frac{1}{2}L)}
\end{aligned}$$

we have the interest rates the lender's would charge having observed the borrower choosing Project i.

(b)

We can compare the borrower's expected returns for Project 1 and 2 to see which the borrower would prefer. We will see that the borrower will prefer the risky project (Project 2) regardless of the lender's choice of interest rate as a result of the default rate being equivalent across the two projects.
By investing in Project 1, the borrower's expected return is

$$E\ \Pi_1 = \frac{1}{2}(R + x_1 - (1+r)L) - \frac{1}{2}(C + R - x_1).$$

The expected return from Project 2 is

$$E\ \Pi_2 = \frac{1}{2}(R + x_2 - (1+r)L) - \frac{1}{2}(C + R - x_2).$$

We assume the above expected returns are equal and then show this is actually false, with the expected return for Project 2 giving the greater return regardless of interest rate.

$$\begin{aligned}
\frac{1}{2}(R + x_1 - (1+r)L) - \frac{1}{2}(C + R - x_1) &= \frac{1}{2}(R + x_2 - (1+r)L) - \frac{1}{2}(C + R - x_2) \\
\frac{1}{2}(R + x_1 - (1+r)L - C - R + x_1) &= \frac{1}{2}(R + x_2 - (1+r)L - C - R + x_2) \\
\frac{1}{2}(2x_1 - (1+r)L - C) &= \frac{1}{2}(2x_2 - (1+r)L - C) \\
x_1 &= x_2 \quad \Rightarrow\!\Leftarrow \\
x_1 &< x_2.
\end{aligned}$$

As $x_2 > x_1$, $E\ \Pi_2$ will always be greater than $E\ \Pi_1$ regardless of r. With rational expectations, the lender (bank) would try to maximise its expected return by charging interest rate

$$r^* = \frac{C + R - x_2 + L}{2 - L}.$$

So, the good project (low risk) will not get funding.

Problem 3

(a)

Assuming perfect competition and no friction, the bank's optimization problem is

$$\max_{a_t, b_t, d_t} R_{a,t}a_t + R_{b,t}b_t - R_{d,t}d_t \quad \text{s.t.} \quad d_t + n_t = a_t + b_t.$$

The Lagrangian is

$$\mathcal{L} = R_{a,t}a_t + R_{b,t}b_t - R_{d,t}d_t + \lambda_t(d_t + n_t - a_t - b_t).$$

The first order conditions are

$$\begin{aligned}
[a_t] &\quad R_{a,t} - \lambda_t = 0 \\
[b_t] &\quad R_{b,t} - \lambda_t = 0 \\
[d_t] &\quad -R_{d,t} + \lambda_t = 0 \\
[\lambda_t] &\quad d_t + n_t - a_t - b_t = 0,
\end{aligned}$$

which can be simplified as $R_{a,t} = R_{b,t} = R_{d,t}$.

(b)

Now, the bank's optimization problem is

$$\max_{a_t,b_t,d_t} R_{a,t}a_t + R_{b,t}b_t - R_{d,t}d_t \quad \text{s.t.} \quad d_t + n_t = a_t + b_t$$
$$R_{a,t}a_t + R_{b,t}b_t - R_{d,t}d_t \geq \theta(a_t + \omega b_t)$$

The Lagrangian is

$$\mathcal{L} = R_{a,t}a_t + R_{b,t}b_t - R_{d,t}d_t + \psi_t(d_t + n_t - a_t - b_t) + \lambda_t(R_{a,t}a_t + R_{b,t}b_t - R_{d,t}d_t - \theta(a_t + \omega b_t)).$$

The first order conditions are

$$\begin{aligned}
[a_t] &\quad R_{a,t} - \psi_t + \lambda_t(R_{a,t} - \theta) = 0 && \Leftrightarrow (1+\lambda_t)R_{a,t} = \psi_t + \lambda_t\theta \\
[b_t] &\quad R_{b,t} - \psi_t + \lambda_t(R_{b,t} - \theta\omega) = 0 && \Leftrightarrow (1+\lambda_t)R_{b,t} = \psi_t + \lambda_t\theta\omega \\
[d_t] &\quad -R_{d,t} + \psi_t - \lambda_t = 0 && \Leftrightarrow (1+\lambda_t)R_{d,t} = \psi_t \\
[\lambda_t] &\quad d_t + n_t - a_t - b_t = 0, \\
[\psi_t] &\quad R_{a,t}a_t + R_{b,t}b_t - R_{d,t}d_t - \theta(a_t + \omega b_t) = 0,
\end{aligned}$$

which can simplified as

$$R_{a,t} - R_{d,t} = \frac{\lambda_t\theta}{1+\lambda_t} \tag{1}$$
$$R_{b,t} - R_{d,t} = \frac{\lambda_t\theta\omega}{1+\lambda_t}, \tag{2}$$

or

$$R_{a,t} - R_{b,t} = \frac{\lambda_t\theta(1-\omega)}{1+\lambda_t}. \tag{3}$$

(c)

From equation 2, the gross returns on bond holdings, $R_{b,t}$ exceeds the rate paid on deposits ($R_{d,t}$) if the incentive constrainst is binding ($\lambda_t > 0$), bank owners divert a fraction of their holdings for their own use ($\theta > 0$) and $\omega = 0$.

From equation 3, the gross returns on assets, $R_{a,t}$ exceeds the gross returns on bond holdings ($R_{b,t}$) if the incentive constrainst is binding ($\lambda_t > 0$), bank owners divert a fraction of their holdings for their own use ($\theta > 0$) and the fraction of asset holdings diverted by banks is less than fraction of bond holdings ($\omega < 1$).

For

$$R_{a,t} > R_{b,t} > R_{d,t}$$

we need $\lambda_t > 0$, $\theta > 0$ and $0 < \omega < 1$.

Problem 4

(a)

If lenders could observe project outcomes costlessly, equilibrium would involve lenders financing all projects whose expected payout exceeds their opportunity cost of $(1 + r) * 1$.

$$1 + r^P + \varepsilon^* = 1 + r$$
$$\varepsilon^* = r - r^P$$

Thus, all firms whose ε is greater than the critical value ε^* would receive loans. ε^* is independent of S. So, whether the project is funded fully by the lender or requires an investment from the borrower, ε^* would not be affected.

(b)

We can consider ε' as the lower threshold beyond which the entire project is taken over by the lender. For $\varepsilon < \varepsilon'$, the lender pays the cost of auditing and borrower does not have the incentive to misreport.
For $\varepsilon > \varepsilon'$, borrower would like to truthfully report the ε if K is independent of ε. If K is dependent on ε, then the borrower would have an incentive to misreport to ε so as to minimize its payment to the lender.

We will prove the above claims by contradiction:
(1) Suppose $\varepsilon < \varepsilon'$ and the borrower tells the lender a value $m(\varepsilon) \geq \varepsilon'$, meaning the borrower will pay the lender KB. From part (a) we know that $1 + r^p + \varepsilon = KB$, which by the assumption we just made about the true ε, implies that $1 + r^p + \varepsilon < KB$. As such, the borrower would prefer to pay $1 + r^p + \varepsilon$, which would require $m(\varepsilon) < \varepsilon'$. Additionally, once the borrower decides to make $m(\varepsilon) < \varepsilon'$, there is no incentive to not be truthful, as the lender will take $1 + r^p + \varepsilon$ regardless of m.

(2) Suppose $\varepsilon \geq \varepsilon'$ and the borrower tells the lender a value $m(\varepsilon) < \varepsilon'$, meaning the borrower will pay the lender $1 + r^p + \varepsilon$. We know, however, that $1 + r^p + \varepsilon \geq 1 + r^p + \varepsilon' = KB$. This would lead the borrower to prefer paying KB instead of $1 + r^p + \varepsilon$, which would require $m(\varepsilon) \geq \varepsilon'$. Since K is independent of ε, the borrower has no incentive to make $m(\varepsilon) \neq \varepsilon$.

(c)

ε' is the threshold that allows the borrower to pay the interest to the borrower so that the entire project is not taken over by the lender. That is,

$$1 + r^P + \varepsilon' = K(1 - S) \quad \Rightarrow \quad \varepsilon' = K(1 - S) - 1 - r^P,$$

which is decreasing in S. In words, the more funds the borrower has, the less investment is required. This leads to a lower threshold, which means that there is more room for the return shock letting the borrower to retain the project.

(d)

The expected payout for the borrower can be expressed as follows:

$$\begin{aligned}
E\Pi^B &= (\frac{1}{2\bar{\varepsilon}}) \int_{\varepsilon'}^{\bar{\varepsilon}} (1 + r^p + \varepsilon - KB) d\varepsilon \\
&= \Big(\frac{1}{2\bar{\varepsilon}}\Big)\Big(\int_{\varepsilon'}^{\bar{\varepsilon}} (1 + r^p - KB) d\varepsilon + \int_{\varepsilon'}^{\bar{\varepsilon}} \varepsilon \; d\varepsilon \Big) \\
&= (1 + r^p - KB)(1 - F(\varepsilon') + \Big(\frac{1}{2\bar{\varepsilon}}\Big) \int_{\varepsilon'}^{\bar{\varepsilon}} \varepsilon \; d\varepsilon \\
&= -\varepsilon'\Big(\frac{1}{2\bar{\varepsilon}}\Big)(\bar{\varepsilon} - \varepsilon') + \Big(\frac{1}{4\bar{\varepsilon}}\Big)(\bar{\varepsilon}^2 - \varepsilon'^2) \\
&= -\varepsilon'\Big(\frac{\bar{\varepsilon} - \varepsilon'}{2\bar{\varepsilon}}\Big) + \Big(\frac{\varepsilon^2 - \varepsilon'^2}{4\bar{\varepsilon}}\Big)
\end{aligned}$$

(e)

The expected payout for the lender can be expressed as follows:

$$\begin{aligned}
E\Pi^L &= \Big(\frac{1}{2\bar{\varepsilon}}\Big) \int_{-\bar{\varepsilon}}^{\varepsilon'} (1 + r^p + \varepsilon - c) d\varepsilon + \Big(\frac{1}{2\bar{\varepsilon}}\Big) \int_{\varepsilon'}^{\bar{\varepsilon}} KB \; d\varepsilon \\
&= (1 + r^p - c)F(\varepsilon') + (\frac{1}{2\bar{\varepsilon}}) \int_{-\bar{\varepsilon}}^{\varepsilon'} \varepsilon \; d\varepsilon + KB(1 - F(\varepsilon')) \\
&= (1 + r^p - c)\Big(\frac{1}{2\bar{\varepsilon}}\Big)(\varepsilon' - \bar{\varepsilon}) + \Big(\frac{1}{4\bar{\varepsilon}}\Big)(\varepsilon'^2 - \bar{\varepsilon}^2) + KB\Big(\frac{1}{2\bar{\varepsilon}}\Big)(\bar{\varepsilon} - \varepsilon') \\
&= (1 + r^p - c)\Big(\frac{1}{2\bar{\varepsilon}}\Big)(\varepsilon' - \bar{\varepsilon}) + \Big(\frac{1}{4\bar{\varepsilon}}\Big)(\varepsilon'^2 - \bar{\varepsilon}^2) + (1 + r^p + \varepsilon')\Big(\frac{1}{2\bar{\varepsilon}}\Big)(\bar{\varepsilon} - \varepsilon') \\
&= (1 + r^p) - c\Big(\frac{1}{2\bar{\varepsilon}}\Big)(\varepsilon' - \bar{\varepsilon}) + \varepsilon'\Big(\frac{1}{2\bar{\varepsilon}}\Big)(\bar{\varepsilon} - \varepsilon') + \Big(\frac{1}{4\bar{\varepsilon}}\Big)(\varepsilon'^2 - \bar{\varepsilon}^2) \\
&= (1 + r^p) - c\Big(\frac{\varepsilon' - \bar{\varepsilon}}{2\bar{\varepsilon}}\Big) + \varepsilon'\Big(\frac{\bar{\varepsilon} - \varepsilon'}{2\bar{\varepsilon}}\Big) + \Big(\frac{\varepsilon'^2 - \bar{\varepsilon}^2}{4\bar{\varepsilon}}\Big)
\end{aligned}$$