ECON 205C: Problem Set 2

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Problem 1

(a)

The central bank's optimization problem under full commitment is

$$\min_{\{\pi_s, x_s, i_s\}_{s=t}^{\infty}} \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda_x x_{t+i}^2] \quad \text{s.t.} \quad \pi_t = (1-\phi)\beta E_t \pi_{t+1} + \phi \pi_{t-1} + \kappa x_t + e_t$$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t).$$

The Lagrangian is

$$\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^i \Big\{ \frac{1}{2}[\pi_{t+i}^2 + \lambda_x x_{t+i}^2] + \psi_{t+i}\big(\pi_{t+i} - (1-\phi)\beta E_t \pi_{t+1+i} - \phi \pi_{t-1+i} - \kappa x_{t+i} - e_{t+i}\big)$$

$$+ \xi_{t+i}\big(x_{t+i} - E_t x_{t+1+i} + \frac{1}{\sigma}(i_{t+i} - \pi_{t+1+i} - r_{t+i})\big) \Big\}.$$

The first order conditions are:

$$\begin{aligned}
&[\pi_t] \quad \pi_t + \psi_t - \beta\phi E_t \psi_{t+1} = 0 \\
&[x_t] \quad \lambda_x x_t - \psi_t \kappa + \xi_t = 0 \\
&[i_t] \quad \frac{1}{\sigma}\xi_t = 0 \\
&[\pi_{t+i}] \quad \beta^i (E_t \pi_{t+i} + E_t \psi_{t+i} - (1-\phi) E_t \psi_{t-1+i} - \beta\phi E_t \psi_{t+1+i} - \frac{1}{\sigma}\xi_{t-1+i} + E_t \xi_{t+i}) = 0 \\
&[x_{t+i}] \quad \beta^i (\lambda_x x_{t+i} - \kappa \psi_{t+i} - \xi_{t-1+i}) = 0 \\
&[i_{t+i}] \quad \frac{1}{\sigma}\xi_{t+i} = 0
\end{aligned}$$

where the first three conditions are for $i = 0$ and the last three are for $i > 0$.
We can see that we can eliminate the Lagrangian multiplier ξ_t and simplify the conditions for $i > 0$ to give us the following:

$$\begin{aligned}
&[\pi_t] \quad \pi_t + \psi_t - \beta\phi E_t \psi_{t+1} = 0 \\
&[x_t] \quad \lambda_x x_t - \psi_t \kappa = 0 \\
&[\pi_{t+i}] \quad E_t \pi_{t+i} + E_t \psi_{t+i} - (1-\phi) E_t \psi_{t-1+i} - \beta\phi E_t \psi_{t+1+i} = 0 \\
&[x_{t+i}] \quad \lambda_x x_{t+i} - \kappa \psi_{t+i} = 0
\end{aligned}$$

where the first conditions are for $i = 0$ and the last two are for $i > 0$.

By further rearrangement, we obtain a targeting rule:

$$\pi_t = \frac{\lambda_x}{\kappa}(\beta\phi E_t x_{t+1} - x_t) \qquad (i = 0)$$

$$\pi_{t+i} = \frac{\lambda_x}{\kappa}(\beta\phi E_{t+i} x_{t+i+1} - x_{t+i} + (1-\phi)x_{t-1+i}) \qquad (i > 0).$$

(b)

From a timeless perspective, there are no separate conditions for $i = 0$. That is, the conditions for π_t and x_t in part (c) are no longer considered. Therefore, the system of first order conditions becomes:

$$[\pi_{t+i}] \quad E_t\pi_{t+i} + E_t\psi_{t+i} - (1-\phi)E_t\psi_{t-1+i} - \beta\phi E_t\psi_{t+1+i} = 0$$

$$[x_{t+i}] \quad \lambda_x x_{t+i} - \kappa\psi_{t+i} + \theta_{t+i} = 0$$

which should be satisfied for all $i \geq 0$. We can combine the two conditions to express them as a targeting rule:

$$\pi_t = \frac{\lambda_x}{\kappa}(\beta\phi E_t x_{t+1} - x_t + (1-\phi)x_{t-1}).$$

(c)

As discussed earlier, the main idea of optimal commitment policy is to mitigate the impact of current shocks through making promises that affect future expectations. Thus, for the policy to be sustainable policy makers should look backward and keep the promises made in the past. The targeting rule in part (b) takes into consideration the lagged output gap from the past and the expected lagged output gap in the future as well. The optimal policy here introduces a forward-looking aspect as we have included the lagged inflation term in the inflation adjustment equation. As much as the policy maker today takes into account yesterday's inflation, today's inflation would have influence on tomorrow's policy decisions. The policy maker should look forward and make expectations about the future economy. This is consistent with having expected future output gap in our targeting rule in part (b).

Problem 2

(a)

The minimized values of the loss function are calculated to be 1.4487 and 0.3539 under optimal discretion and under optimal commitment, respectively.

(b)

Table 1 displays the values of the loss function for different levels of inertia in the central bank's policy rule.

Table 1: The value of the loss function

ρ_i	L	σ_x^2	σ_π^2
0	0.1713	0.0047	0.0005
0.25	0.1661	0.0046	0.0005
0.50	0.1806	0.0044	0.0007
0.75	0.4228	0.0044	0.0007

(c)

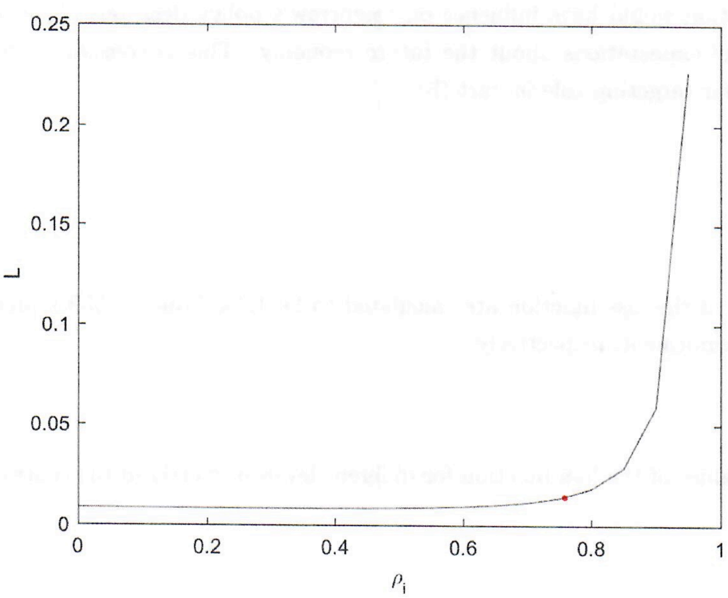
Table 2: The value of the loss function setting ρ_r and ρ_u to zero

ρ_i	L	σ_x^2	σ_π^2
0	0.0086	4.2259×10^{-5}	7.5127×10^{-5}
0.25	0.0083	4.2255×10^{-5}	7.2042×10^{-5}
0.50	0.0085	5.8770×10^{-5}	7.0477×10^{-5}
0.75	0.0137	1.7861×10^{-4}	9.2165×10^{-5}

$-\frac{1}{2}$ When ρ_i is high, the central bank captures more inertia into the policy rule. When it is close to zero, the policy behaves in a similar fashion as a optimal commitment where the loss is minimized.

but commitment adds inertia

Figure 1: The relationship between ρ_i and the value of the loss function



Problem 3

(a)

We will begin by defining $E_t\pi_{t+1}$ and $E_t x_{t+1}$ as follows:

$$E_t\pi_{t+1} = (1-q)(0) + q\pi^{zlb} = q\pi^{zlb}$$
$$E_t x_{t+1} = (1-q)(0) + qx^{zlb} = qx^{zlb}.$$

By plugging these values into the given model and replacing the normal variables with *zlb* variables we get the following equations:

$$\pi^{zlb} = \beta q\pi_{zlb} + \kappa x^{zlb}$$
$$x^{zlb} = qx^{zlb} + \frac{1}{\sigma}[q\pi^{zlb} + \bar{r} + r^{zlb}],$$

which can be written in matrix form as:

$$\begin{bmatrix} 1-\beta q & -\kappa \\ \frac{-q}{\sigma} & 1-q \end{bmatrix} \begin{bmatrix} \pi^{zlb} \\ x^{zlb} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{\sigma}(\bar{r} + r^{zlb}) \end{bmatrix}$$

$$\begin{aligned}
\Leftrightarrow \begin{bmatrix} \pi^{zlb} \\ x^{zlb} \end{bmatrix} &= \begin{bmatrix} 1-\beta q & -\kappa \\ \frac{-q}{\sigma} & 1-q \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{1}{\sigma}(\bar{r} + r^{zlb}) \end{bmatrix} \\
&= \left((1-\beta q)(1-q) - \frac{\kappa}{\sigma}q\right)^{-1} \begin{bmatrix} 1-q & \kappa \\ \frac{q}{\sigma} & 1-\beta q \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{\sigma}(\bar{r} + r^{zlb}) \end{bmatrix} \\
&= \left((1-\beta q)(1-q) - \frac{\kappa}{\sigma}q\right)^{-1} \begin{bmatrix} \frac{\kappa}{\sigma}(\bar{r} + r^{zlb}) \\ \frac{1}{\sigma}(1-\beta q)(\bar{r} + r^{zlb}) \end{bmatrix} \\
&= \left(\sigma(1-\beta q)(1-q) - \kappa q\right)^{-1} \begin{bmatrix} \kappa(\bar{r} + r^{zlb}) \\ (1-\beta q)(\bar{r} + r^{zlb}) \end{bmatrix}.
\end{aligned}$$

(b)

An increase in q indicates there is higher probability that the zero lower bound period will continue. This means that the expectations shift over a longer horizon and the jump of the output gap becomes larger. A fall in expected inflation increases the real interest rate, which makes the consumers postpone their spending, lowering the current aggregate demand and the output gap falls.

(c)

$i_t = 0$, $x_t < 0$ and $\pi_t = 0$ are not consistent with the results in part (a) because x_t and π_t should have the same sign as long as $u_t = 0$ for all t. If $u_t \neq 0$, a positive inflation shock may increase π_t without directly affecting x_t, which would be consistent with $x_t < 0$ and $\pi_t = 0$.

Problem 4

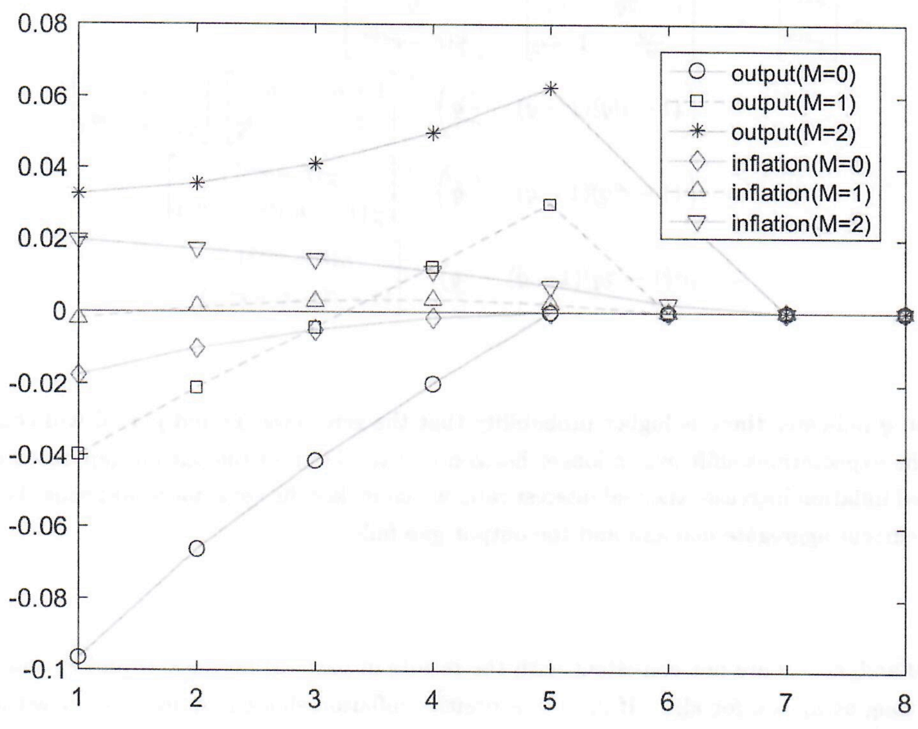
We can see from the system of equations we found in part (a) of Problem 3 that κ has a positive relationship with the scalar multiplying the right side vector. Since $\bar{r} + r^{zlb} < 0$, we know that an increase in κ would force x^{zlb} to become more negative. Similarly an increase in κ would push π^{zlb} further negative. A higher κ implies a lower degree of nominal rigidity, which means we have more flexible prices. κ signifies the elasticity of inflation with respect to output gap. In a flexible price economy, the effect on fall in inflation would be larger. And if it is persistent the expected inflation would also fall more than in the economy with sticky prices. As consumers expect falling prices, they postpone their spending, leading to a fall in aggregate demand and output gap, which further lowers inflation. So an economy with flexible prices will show a greater fall in inflation as well as output gap. As such, the economy boasting more flexible prices would have a larger (negative) output gap and a lower (more negative) inflation rate than the one with stickier prices.

Problem 5

(a)

Figure 2 plots the equilibrium for output gap and inflation by different values of M.

Figure 2: Equilibrium for output gap and inflation



(b)

Forward guidance, that is, promising to keep future interest rates at zero, is a powerful tool in boosting output. As evident in Figure 2, the jump in inital output gap increases as M moves from 0 to 2. By promising to keep interest rates low even after the economy exits a zero lower bound state, the economy experiences a boom. The further into the future this promise is said to persist, the greater is the jump in the initial output gap.