# 205C Final Exam Preparation

David Sungho Park

June 25, 2017

1. Discuss the effects of a positive aggregate productivity shock on output, the output gap, employment and real wages in each of the following three models:

   (a) A real business cycle model;

   $$z_t \uparrow \;\Rightarrow y_t \uparrow \Rightarrow c_t \uparrow \Rightarrow \textit{labor} \downarrow$$
   $$\Rightarrow w_t \uparrow \Rightarrow MPL \uparrow \Rightarrow \textit{employment} \uparrow$$

   no output gap.

   (b) A new Keynesian model with sticky prices (assume an optimal discretionary monetary policy);

   $$z_t \uparrow \Rightarrow y_t^f \uparrow \Rightarrow w_t \uparrow \text{ such that } x_t = y_t - y_t^f = 0 \;\Rightarrow y_t \uparrow$$

   (c) A new Keynesian model with sticky prices and wages (assume an optimal discretionary monetary policy).

   $$z_t \uparrow \Rightarrow y_t^f \uparrow \Rightarrow x_t < 0$$

   $w_t$ cannot adjust to keep $x_t = 0$. $y_t$ increases but not as much as $y_t^f$ does. So $x_t < 0$.

2. [Again!] Suppose the economy is represented by the following very simple two equation system:

   $$i_t = r_t + \mathrm{E}_t \pi_{t+1}$$

   $$i_t = \max(r + \pi^* + \delta(\pi_t - \pi^*), 0),$$

   where the assumption of perfect foresight has been imposed. The notation is $i$ and $r$ for the nominal and real interest rates, $\pi$ for inflation and $\pi^*$ for the central bank's inflation target. $r$ is the steady state real interest rate. Assume $r_t$ follows an exogenous process: $r_t = r + \rho(r_{t-1} - r) + e_t$ where $r > 0$, $0 < \rho < 1$, and $e_t$ is mean zero i.i.d. process. Also assume $\delta > 1$. Suppose attention is restricted to equilibria in which $i_t > 0$. Eliminate $i_t$ to obtain a single equation for inflation. Does this equation satisfy the Blanchard-Kahn conditions? Explain.

The two equations can be simplified into one:

$$\begin{aligned} r_t + \mathrm{E}_t\pi_{t+1} &= \max(r + \pi^* + \delta(\pi_t - \pi^*), 0) \\ \implies r_t + \mathrm{E}_t\pi_{t+1} &= r + \pi^* + \delta(\pi_t - \pi^*) \\ \iff \delta\pi_t &= r_t + \mathrm{E}_t\pi_{t+1} - r - \pi^* + \delta\pi^* \\ \iff \pi_t &= \frac{1}{\delta}(r_t + \mathrm{E}_t\pi_{t+1} - r - \pi^*) + \pi^* \end{aligned}$$

3. Suppose the economy's inflation rate is describe by the following equation (all variable expressed as percentage deviations around a zero inflation steady state):

$$\pi_t = \beta \mathrm{E}_t\pi_{t+1} + \kappa x_t + e_t,$$

where $x_t$ is the gap between output and the flexible-price equilibrium output level, and $e_t$ is a cost shock. Assume that

$$e_t = \rho_e e_{t-1} + \varepsilon_t,$$

where $\psi$ and $\varepsilon$ are white noise process. The central bank sets the nominal interest rate $i_t$ to minimize

$$\frac{1}{2}\mathrm{E}_t \sum_{i=1}^{\infty} \beta_i(\pi_{t+i}^2 + \lambda x_{t+i}^2).$$

(a) Derive the first-order conditions linking inflation and the output gap for the *fully* optimal commitment policy.

The central bank's optimization problem under commitment is

$$\min_{\{\pi_s, x_s\}_{s=t}^{\infty}} \frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i((\pi_{t+i}^2 + \lambda x_{t+i}^2)) \quad \text{s.t.} \quad \pi_t = \beta \mathrm{E}_t\pi_{t+1} + \kappa x_t + e_t.$$

The Lagrangian is

$$\mathcal{L} = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left( \frac{1}{2}(\pi_{t+i}^2 + \lambda x_{t+i}^2) + \psi_{t+i}(\pi_{t+i} - \beta \mathrm{E}_{t+i}\pi_{t+i+1} - \kappa x_{t+i} - e_{t+i} \right).$$

The FOCs are

$$\begin{aligned} &[\pi_t] \quad \pi_t + \psi_t = 0 && (i = 0) \\ &[x_t] \quad \lambda x_t - \psi_t \kappa = 0 && (i = 0) \\ &[\pi_{t+i}] \quad \pi_{t+i} + \psi_{t+i} - \mathrm{E}_{t+i}\psi_{t+i-1} = 0 && (i \geq 1) \\ &[x_{t+i}] \quad \lambda x_{t+i} - \psi_{t+i}\kappa = 0 && (i \geq 1). \end{aligned}$$

$$\iff$$

(b) Explain why the first-order condition for time $t$ differs from the first-order condition for $t + i$ for

$i > 0$.

From the inflation adjustment equation, a central bank, faced with a cost shock, can better stabilize current inflation if it can adjust both the current output gap ($x_t$) and the private sector's expectations about future inflation ($\mathrm{E}_t\pi_{t+1}$). Or, since the central bank cares about both output gap and inflation stabilization, we can express this by saying that if the cost shock is positive, a given rise in inflation can be achieved with a smaller decline in the output gap if expected future inflation is reduced. Thus, the optimal commitment policy will promise a deflation in period $t+1$ so that $E_t\pi_{t+1} < 0$. This promise must be fulfilled, so at time $t+1$ the central bank's actions reflect promises made at time $t$, making policy backward looking. At time $t$, there was no past period in which promises had been made, so the CB's optimal policy only depends on the current state of the economy.

(c) What is meant by a commitment policy that is optimal from a timeless perspective? (Explain in words.)

The policy should be backward looking at all times. That is, the special nature of the first period, when the effects on the past can be ignored, cannot be exploited.

(d) What is the first-order condition linking inflation and the output gap that the central bank follows under an optimal commitment policy from a timeless perspective?

From the FOCs derives in part (b), we only take the equations for $i \geq 1$.

$$\begin{aligned}
&\pi_{t+i} + \psi_{t+i} - \mathrm{E}_t\psi_{t+i-1} = 0 \\
&\lambda x_{t+i} - \psi_{t+i}\kappa = 0 \\
\implies\ &\pi_{t+i} + \frac{\lambda x_{t+i}}{\kappa} - \mathrm{E}_{t+i}\frac{\lambda x_{t+i+1}}{\kappa} = 0 \\
\iff\ &\pi_{t+i} = \frac{\lambda}{\kappa}\mathrm{E}_{t+i}[x_{t+i+1} - x_{t+i}].
\end{aligned}$$

(e) Explain why, under commitment, the central bank promises a deflation in the period after a positive cost shock (assume the cost shock is serially uncorrelated).

Faced with a positive cost shock ($e_t > 0$), a central bank can stabilize current inflation by adjusting the private sector's expectations about future inflation to be negative ($\mathrm{E}_t\pi_{t+1} < 0$). Thus, the optimal commitment policy will promise a deflation in period $t+1$ so that $E_t\pi_{t+1} < 0$.

(f) In the face of cost shocks, explain why the optimal commitment policy achieves a better trade off between inflation and output gap stability than is achieved under optimal discretion.

From the inflation adjustment equation, a central bank, faced with a positive cost shock, can better stabilize current inflation if it can adjust both the current output gap ($x_t$) and the private sector's expectations about future inflation ($\mathrm{E}_t\pi_{t+1}$). Or, since the central bank cares about both output gap and inflation stabilization, we can express this by saying that if the cost shock is positive, a given rise in inflation can be achieved with a smaller decline in the output gap if expected future inflation is reduced.

4. Consider a basic new Keynesian model with Calvo adjustment of prices and flexible nominal wages.

(a) [Again!] In the absence of cost shocks, optimal policy would ensure inflation and the output gap both remain equal to zero so that actual output moves with the flex-price output level. In turn, the flex-price output level depends on shocks to productivity. Explain why output can fluctuate

to keep the output gap at zero in response to productivity shocks even though prices are sticky.
Even with sticky prices, if wages are flexible real wages can adjust to ensure labor market equilibrium in face of productivity shocks.

(b) Suppose both prices and nominal wages are sticky (assume a Calvo model for both prices and wages). Is it still possible for the central bank to maintain zero inflation and a zero output gap in the face of productivity shocks? Explain.
No. If both prices and wages are sticky, then the real wage will not be able to adjust to match what it would do in the flex-price/wage equilibrium. Monetary policy can stability prices (wages), but then wage inflation (price inflation) and the output gap will move in the face of shocks since the real wage doesn't jump to ensure the output gap remaining at zero. With two nominal rigidities, one policy instrument isn't sufficient.

5. Explain why inflation is costly in a new Keynesian model.
Inflation with sticky prices leads to price dispersion ($\Delta_t > 1$), since firms that can adjust prices will adjust their prices taking into account expected future MC. Such price dispersion leads to an increase in required labor to produce a fixed level of $C_t$ compared to a flex-price labor where $\Delta_t = 1$ (i.e., $N_t = \frac{C_t}{z_t}\Delta_t > N_t^f$). Otherwise, in the case the labor is fixed, we get lower level of $C_t$. Since working more or consuming less generates disutility, household's have welfare loss.

6. Suppose a sticky-price new Keynesian model is given by

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

If the central bank wants to ensure $x_t = \pi_t = 0$, what problems might arise if the central bank sets its interest rate instrument according to the rule $i_t = r_t^n$?
The problem is that this doesn't ensure a stationary or unique equilibrium. When $i_t = r_t^n$, $i_t$ does not respond to the endogenous variables $x_t$ and $\pi_t$. So the Taylor principle is not satisfied.
For example, suppose $\mathrm{E}_t \pi_{t+1}$ increases. Then $r_t (= i_t - \mathrm{E}_t \pi_{t+1})$ will decrease since $i_t$ cannot be adjusted, which leads to an increase in $x_t$ due to expansionary effects. By the PC, $x_t \uparrow$ leads to $\pi_t \uparrow$; this leads the economy into an explosive path. So the $x_t = \pi_t = 0$ equilibrium will not be stationary or unique.

7. Consider the following simple NK model:

$$y_t = \mathrm{E}_t y_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - \rho_t)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa(\sigma + \eta)(y_t - y_t^f),$$

where $y$ is *output* (i.e., not an output gap) and $y^f$ is the flex-price output level, both defined as percent deviations around the steady state, while $i$ is the nominal interest rate and $\pi$ is inflation, both expressed as deviations around their steady state value of $\beta^{-1} - 1$ and 0 respectively. And $\rho_t$ is a stochastic shock to the steady state discount rate $\beta$. Assume $\rho_t$ follows the following exogenous process: if $\rho_{t+s} = \rho > 0$ for some $s > 0$, then $\rho_{t+s+i} = \rho$ for all $i \geq 0$. If $\rho_t = \rho$, then $\rho_{t+1} = -\rho$ with probability $\mu$ and $\rho$ with probability $1 - \mu$. When $\rho_t = -\rho$, $i_t = 0$. Suppose for $s > 0$ whenever $\rho_{t+s} = \rho, i_{t+s} = \rho_{t+s}, \pi_{t+s} = 0$ and $y_{t+s} = y_{t+s}^f$. Finally, assume $y_t^f = \gamma z_t$, where $z_t$ is a mean zero, i.i.d. process.

(a) Suppose the economy starts out with $\rho_t = -\rho$. Find the equilibrium values of $\pi_t$ and $y_t$.

Since $\rho_t = -\rho$, we have

$$
\begin{aligned}
i_t &= 0 \quad \text{(i.e. ZLB)} \\
\rho_{t+1} &= \begin{cases} -\rho & (p = \mu) \\ \rho & (p = 1-\mu) \end{cases}
\end{aligned}
$$

At ZLB, $x_t = x^z, \pi_t = \pi^z$; once out of ZLB, $x_t = \pi_t = 0, i_t = \rho_t$. Therefore,

$$
\begin{aligned}
\mathrm{E}_t x_{t+1} &= \mu x^z + (1-\mu)0 = \mu x^z \\
\mathrm{E}_t \pi_{t+1} &= \mu \pi^z + (1-\mu)0 = \mu \pi^z
\end{aligned}
$$

Our first equation of the given NK model can be expressed in terms of output gap:

$$
\begin{aligned}
y_t &= \mathrm{E}_t y_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - \rho_t) \\
\iff y_t - y_t^f &= \mathrm{E}_t y_{t+1} - \mathrm{E}_t y_{t+1}^f - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - \rho_t) + \mathrm{E}_t y_{t+1}^f - y_t^f \\
\iff y_t - y_t^f &= \mathrm{E}_t y_{t+1} - \mathrm{E}_t y_{t+1}^f - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - \rho_t - \sigma \mathrm{E}_t y_{t+1}^f + \sigma y_t^f) \\
\iff x_t &= \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - (\rho_t + \sigma(\mathrm{E}_t y_{t+1}^f - y_t^f)) \\
\iff x_t &= \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - \rho_t).
\end{aligned}
$$

So evaluating at the ZLB case,

$$
\begin{aligned}
x^z &= \mu x^z - \frac{1}{\sigma}(0 - \mu \pi^z + \rho) \\
\pi^z &= \beta \mu \pi^z + \kappa(\sigma + \eta) x^z,
\end{aligned}
$$

which can be solved as

$$
\begin{aligned}
\pi^z = \frac{\sigma(1-\mu)x^z + \rho}{\mu} = \frac{\kappa(\sigma+\eta)x^z}{1-\beta\mu} &\iff (1-\beta\mu)(\sigma(1-\mu)x^z + \rho) = \mu\kappa(\sigma+\eta)x^z \\
&\iff x^z = \frac{(1-\beta\mu)\rho}{\mu\kappa(\sigma+\eta) - (1-\beta\mu)\sigma(1-\mu)} \\
&\implies \pi^z = \frac{\kappa(\sigma+\eta)\rho}{\mu\kappa(\sigma+\eta) - (1-\beta\mu)\sigma(1-\mu)}
\end{aligned}
$$

Note that both $x^z$ and $\pi^z$ are negative, since $\mu\kappa(\sigma+\eta) - (1-\beta\mu)\sigma(1-\mu) < 0$.

(b) *Explain* how the values you obtained in part (a) are affected if $\mu$ increases.
An increase in $\mu$ means there is more pessimism that the economy is more likely to stay in negative shock. Since from part (a) $x^z$ and $\pi^z$ are negative, a higher value of $\mu$ decreases $\mathrm{E}_t x_{t+t}$ and $\mathrm{E}_t \pi_{t+1}$. If the shock is persistent, $\mu \uparrow \Rightarrow \mathrm{E}_t x_{t+1} \downarrow \Rightarrow x^z \downarrow$; and $\mu \uparrow \Rightarrow \mathrm{E}_t \pi_{t+1} \downarrow \Rightarrow r_t \uparrow$ ($\because$ $i_t = 0$) $\Rightarrow x_t \downarrow$ ($\because$ contractionary effect). And again, a decrease in output gap leads to a decrease in inflation by PC.

(c) If $\rho_t = -\rho$, how are $\pi_t$ and $y_t$ affected by a positive productivity shock (i.e., $z_t > 0$)? *Explain what is going on.*
With a positive productivity shock, the flex-price output ($y_t^f$) increases. If the central bank wants

to stabilize the output gap ($x_t$), it has to increase the output level ($y_t$). But since the economy is at the zero lower bound, nominal interest rate ($i_t$) is cannot decrease any further so it can only increase output by increasing expected future inflation ($E_t\pi_{t+1}$). In such case, $y_t$ can increase and $\pi_t$ is not changed. However, if CB cannot affect $E_t\pi_{t+1}$, then output gap drops and inflation rate also drops accordingly by PC.

8. Assume the utility function of the representative household in a small open economy is

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln(C_t) - \frac{N_t^{1+\eta}}{1+\eta} + \frac{a_m}{1-\gamma_m}\left(\frac{M_t}{P_t}\right)^{1-\gamma_m} \right\},$$

where $C$ is total consumption, $N$ is labor supply, and $M/P$ is real money holdings.

$$C_t = \left[(1-\gamma)^{\frac{1}{a}}(C_t^h)^{\frac{a}{a-1}} + \gamma^{\frac{1}{a}}(C_t^f)^{\frac{a}{a-1}}\right]^{\frac{a}{a-1}} \quad , a > 1,$$

and utility is maximized subject to the sequence of constraints given by

$$P_tC_t + M_t + e_t\frac{1}{1+i_t^*}B_t^* + \frac{1}{1+i_t}B_t \le W_tN_t + M_{t-1} + e_tB_{t-1}^* + B_{t-1} + \tau_t,$$

where $B^*$ denotes foreign currency bonds, $e$ is the nominal exchange rate (\$/¥). The nominal interest rate on domestic (foreign bonds is $i(i^*)$ and $\tau$ are lump-sum transfers. Let $P_t^h(P_t^f)$ be the average price of domestically (foreign) produced consumption goods.

(a) Derive the First-order conditions for the household's problem.

(b) Show that the choice of home-produced consumption goods relative to foreign-produced consumption goods depends on the terms of trade.

(c) Derive an expression for the CPI price index $P_t$.

9. If $V_{t,t+1}(s)$ is the price of a claim that pays one unit of domestic currency at $t+1$ in state $s$, $\tilde{p}(s)$ is the probability of state $s$, and $C_t^{-\sigma}$ is the marginal utility of consumption, explain why we expect

$$\left(\frac{V_{t,t+1}(s)}{P_t}\right)C_t^{-\sigma} = \tilde{p}_{t+1}(s)\beta\left(\frac{1}{P_{t+1}(s)}\right)C_{t+1}^{-\sigma} \tag{1}$$

to hold.

(a) If $S_t$ is the nominal exchange rate (price of foreign currency in terms of domestic currency) and $P_t$ is the foreign price index, what parallel condition should hold if foreign residents can also purchase the same state contingent claim?

(b) Show that this condition, together with (1) implies

$$C_t = vQ_t^{\frac{1}{\sigma}}C_t^*$$

where $Q_t \equiv S_t P_t^*/P_t$ is the real exchange rate and $v$ is a constant.

(c) Use these results to obtain the uncovered interest rate parity condition. Can you provide economic intuition to explain this equation?

10. A simple NK small open economy model can be expressed as

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma_\gamma}\right)(i_t - \mathrm{E}_t \pi_{t+1} - \rho_t)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa_\gamma x_t + e_t.$$

The model would then be closed with a specification of monetary policy. While this looks like a NK closed economy model, it is not identical. How does the open economy model given by these two equations differ from the closed economy NK model?

(1) The elasticity of demand w.r.t. the real interest rate is no longer equal to the elasticity of intertemporal substitution ($\frac{1}{\sigma}$). Instead, it is

$$\frac{1}{\sigma_\gamma} = \frac{1 - \gamma(1 - \phi)}{\sigma},$$

which depends on the openness of the economy, $\gamma$.

(2) The real interest rate depends on the development of the world.

$$\rho_t = \rho^* + \sigma_\gamma \mathrm{E}_t(\tilde{y}_{t+1} - \tilde{y}_t)$$

where $\tilde{y}_t = [(\sigma - \sigma_\gamma)y_t^* - (1 + \eta)a_t]/(\eta + \sigma_\gamma)$

(3) $\kappa_\gamma$ also depends on $\sigma_\gamma$

$$\kappa_\gamma = \kappa(\eta + \sigma_\gamma)$$

11. Consider the model of a small open economy (SOE) discussed in lecture. Suppose it is hit by a negative domestic productivity shock.

(a) How are the output gap and cpi inflation affected if the central bank tries to stabilize domestic price inflation? What happens to the terms of trade and the nominal exchange rate. Explain.

(b) How are the output gap and cpi inflation affected if the central bank tries to stabilize the nominal exchange rate? What happens to the terms of trade and domestic price inflation. Explain.

See Problem 25.

12. In the Diamond-Dybvig model of bank runs, why are banks able to create liquidity? How does this make them vulnerable to runs?

See Problem 2 of Part B.

13. Provide a simple example to illustrate why a borrower and a lender might rank risky projects differently, with the borrower preferring the more risky project. Explain the intuition.

Suppose there are two projects: project 1 (risky) and project 2 (less risky); and two states: good and bad. In good state, the riskier project has higher return than the less risky ($x_{1,g} > x_{2,g}$); yet in bad state, $x_{1,b} < x_{2,b}$. Suppose the borrower pays the lender a fixed amount $\bar{k}$ in good state, and the lender

takes over the project in bad state and gets return $x_b < \bar{k}$.
Lender: The lender gets $\bar{k}$ in good state no matter what project, and gets $x_{1,b}$ or $x_{2,b}$ in bad state depending on the project. Since $x_{1,b} < x_{2,b}$, the lender prefers the less risky project.
Borrower: The borrower gets 0 in bad state no matter what project, and gets $(x_{1,g} - \bar{k})$ or $(x_{2,g} - \bar{k})$ in good state depending on the project. Since $x_{1,g} > x_{2,g}$, the borrower prefers the risky project.

14. Assume firms observe the outcome of their own investment projects costlessly; lenders (banks) must incur a monitoring cost to observe project outcomes. Firms and lenders are assumed to be risk neutral. Firms are indexed by efficiency type $\omega$, distributed uniformly on [0,1]. Projects themselves require inputs of $x(\omega)$, with $x' > 0$, yielding gross payoff $\kappa_1$ with probability $\pi_1$ and $\kappa_2 > \kappa_1$ with probability $\pi_2 = q - \pi_1$. The expected project return is $\kappa \equiv \pi_1\kappa_1 + \pi_2\kappa_2$. Assume $x(\omega) = 0.8 \times (\omega + .95)^2$, $\kappa_1 = 1.01, \kappa_2 = 1.4$ and $\pi_1 = \pi_2 = 1/2$ so $\kappa = 1.205$. The realized outcome of a particular project can be observed costlessly by the firm undertaking the project and by the bank at cost $c = 0.025$. Firms are assumed to have internal sources of financing equal to $S < x(0) = 0.72$, so that even the most efficient firm must borrow to undertake a project. Finally, let $r$ denote the opportunity cost of funds to lenders; firms that do not undertake a project also receive this rate on their funds. Assume $r = 1.15$.

    (a) If lenders could observe project outcomes costlessly, all firms whose $\omega$ is less than a critical value $\omega^*$ would obtain funding. Solve for $\omega^*$.
    All firms obtain funding, if expected payoff = opportunity cost

$$\kappa = r \cdot x(\omega^*)$$

$$1.205 = 1.15 \cdot (0.8 \times (\omega^* + .95)^2)$$

$$\omega^* = \sqrt{1.205/(1.15 \cdot 0.8)} - .95$$

    (b) Find the function $S^*(\omega)$ such that a firm of type $\omega$ with internal funds $S \geq S^*(\omega)$ would never need to be audited.
    No auditing is required, if the borrower can always repay even if under the bad state.

$$\kappa_1 = rB = r(x(\omega) - S^*(\omega))$$

$$S^*(\omega) = x(\omega) - \frac{\kappa_1}{r}$$

    (c) With imperfect information, let $p$ be the probability that the firm is audited (i.e., the lender pays the monitoring cost to observe the true outcome) when the firm announces $\kappa_1$. What is the condition that must be satisfied to ensure the firm truthfully reports the project outcome?
    Truthfulness requires the payoff to the firm in the good state $\kappa_2 - rB$ is greater than the expected payoff if the firm reports $\kappa_1$ when it is actually $\kappa_2$.

$$\kappa_2 - rB \geq (1 - p)(\kappa_2 - \kappa_1)$$

    (d) Solve for the optimal lending contract that maximizes the expected payoff to the firm, subject to the requirement that the lender's expected return must be at least as great as her opportunity cost

$rB = r[x(\omega) - S]$ and the firm must have no incentive to report the bad state when in fact the good state occurred. What is the optimal auditing probability? Plot this probability as a function of $\omega$ for $S = 0.2$. Repeat for $S = 0.4$. Since $p \in [0, 1]$, interpret your results.

$$\pi_1(\kappa_1 - pc) + \pi_2(\kappa_2 - (1-p)(\kappa_2 - \kappa_1)) = rB = r[x(\omega) - S]$$

15. Consider a bank with assets $A_t + B_t$ and liabilities $D_t$, where $A$ represents loans, $D$ deposits and $B$ holdings of government bonds. Let $N$ be bank capital, so the bank's balance sheet is $A_t + B_t = D_t + N_t$. Much like the farmers in Kiyotaki and Moore who can remove their labor from the land, assume the bank owners can potentially remove some of the bank's assets for their own use and let the bank fail. Specifically, assume they can divert a fraction $\theta$ of $A_t + \omega B_t$, where $0 \leq \omega \leq 1$. If $\omega = 0$, then none of the bank's holdings of government bonds can be diverted, while if $\omega = 1$, then bond holdings are just as subject to being diverted as loans. Let $V$ equal the continuation value of the bank (i.e., the present discounted value of profits the bank earns by remaining in business).

    (a) Explain in words why depositors will only provide funds to the bank as long as $V_t \geq \theta(A_t + \omega B_t)$. What would happen if $V_t < \theta(A_t + \omega B_t)$?
    Suppose $V_t < \theta(A_t + \omega B_t)$. If the bank owners remove $\theta(A_t + \omega B_t)$ and the bank fails, the depositors will be left with $(1-\theta)(A_t + \omega B_t) + V_t$ , but

    $$(1-\theta)(A_t + \omega B_t) + V_t < A_t + \omega B_t \leq A_t + B_t = D_t + N_t.$$

    That is, the depositors will be left with less amount than what they deposited. Therefore, the depositors will provide funds to the bank only if $V_t \geq \theta(A_t + \omega B_t)$.

    (b) Assume $V_t = v_{a,t}A_t + v_{b,t}B_t - v_{d,t}D_t$ where $R_{a,t}$ is the gross return on assets, $R_{b,t}$ is the gross return on bonds, and $R_{d,t}$ is the gross cost of deposits. Given $N_t$, the bank's problem is to

    $$\max_{A,B,D} R_{a,t}A_t + R_{b,t}B_t - R_{d,t}D_t$$

    subject to

    $$N_t \geq A_t + B_t - D_t$$

    and

    $$v_{a,t}A_t + v_{b,t}B_t - v_{d,t}D_t \geq \theta(A_t + \omega B_t).$$

    Letting $\lambda_t$ be the Lagrangian multiplier on the balance sheet constraint and $\phi_t$ the multiplier on the incentive constraint $V_t \geq \theta(A_t + \omega B_t)$, what are the first order conditions for the bank's choice of $A_t, B_t$ and $D_t$ after $\lambda_t$ is eliminated?
    The Lagrangian is

    $$\mathcal{L} = R_{a,t}A_t + R_{b,t}B_t - R_{d,t}D_t + \lambda_t(N_t - A_t - B_t + D_t) + \phi_t(v_{a,t}A_t + v_{b,t}B_t - v_{d,t}D_t - \theta(A_t + \omega B_t)).$$

The FOCs are

$$
\begin{aligned}
&[A_t] \quad R_{a,t} - \lambda_t + \phi_t(v_{a,t} - \theta) = 0 \\
&[B_t] \quad R_{b,t} - \lambda_t + \phi_t(v_{b,t} - \theta\omega) = 0 \\
&[D_t] \quad - R_{d,t} + \lambda_t - \phi_t v_{d,t} = 0,
\end{aligned}
$$

By eliminating $\lambda_t$,

$$R_{a,t} + \phi_t(v_{a,t} - \theta) = R_{b,t} + \phi_t(v_{b,t} - \theta\omega) = R_{d,t} + \phi_t v_{d,t}.$$

(c) Suppose the constraint $V_t \geq \theta(A_t + \omega B_t)$ is not binding so that $\phi_t = 0$. What can you say about the interest rate spreads $R_{a,t} - R_{b,t}$, $R_{a,t} - R_{d,t}$, and $R_{b,t} - R_{d,t}$?
If $\phi_t = 0$, then $R_{a,t} = R_{b,t} = R_{d,t}$. There is zero interest spread. If the incentive constraint is not binding, banks can always raise deposit to arbitrage the difference between $R_{a,t}$ and $R_{d,t}$. At equilibrium, the arbitrage would ensure interest rate spreads are equal to zero.

(d) Suppose the constraint $V_t \geq \theta(A_t + \omega B_t)$ is just binding. Now what can you say about the interest rate spreads? If $R_{a,t} > R_{d,t}$, why can't the bank arbitrage away this difference by raising more deposit funds and expanding their loans?
Now, $V_t = \theta(A_t + \omega B_t)$.
Since $R_{a,t} > R_{d,t}$ means that return on bank loans is higher than the cost of deposits, the banks wants to raise more deposits and invest in loans. However, since the incentive contraint is binding, their ability to borrow is limited.

(e) Suppose the constraint $V_t \geq \theta(A_t + \omega B_t)$ is binding and $\phi_t > 0$. Can the central bank relax this constraint by selling government bonds to the bank and buying some of the banks loans? Explain.
Now, $V_t = \theta(A_t + \omega B_t)$ and $\phi_t > 0$.
In this case, banks' borrowing in the private sector is limited. Yet banks' borrowing constraint can be relaxed if the CB buys banks' loan financed by selling government bonds (i.e. $B_t \uparrow$ and $A_t \downarrow$) as $0 < \omega < 1$.

16. Consider a standard new Keynesian model with sticky prices and wages; both adjust according to a simple Calvo model but with different degrees of stickiness.

(a) What are the driving variables for price inflation and wage inflation?
The equation for price inflation (or NKPC) can be solved forward as

$$
\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} + \kappa_p \,\widehat{mc}_t \\
&= E_t\Big[\beta^2 \pi_{t+2} + \kappa_p \beta \,\widehat{mc}_{t+1} + \kappa_p \,\widehat{mc}_t\Big] \\
&= E_t\Big[\beta^3 \pi_{t+3} + \kappa_p \,\widehat{mc}_{t+2} + \kappa_p \beta \,\widehat{mc}_{t+1} + \kappa_p \,\widehat{mc}_t\Big] \\
&\;\;\vdots \\
&= \kappa_p E_t \sum_{i=0}^{\infty} \beta^i \,\widehat{mc}_{t+i} \\
&= \kappa_p E_t \sum_{i=0}^{\infty} \beta^i \,(\widehat{\omega}_{t+i} - \widehat{mpl}_{t+i}),
\end{aligned}
$$

where $\widehat{mc}_t$ denotes the real marginal cost in terms of deviation from the flexible-price equilibrium. Since the real marginal cost can be expressed as $\widehat{mc}_t = \widehat{\omega}_t - \widehat{mpl}_t$, we can see the driving variable of price inflation is $(\widehat{\omega}_t - \widehat{mpl}_t)$. where $\omega_t$ and $mpl_t$ denote real wage and marginal product of labor, respectively. Firms wish to set their price relative to the general level of prices as a markup over real marginal cost. So if real marginal cost rises, firms that can adjust will raise their price. Similarly, the equation for wage inflation can be solved forward as

$$\begin{aligned}
\pi_t^w &= \beta E_t \pi_{t+1}^w + \kappa_w(\widehat{mrs}_t - \widehat{\omega}_t) \\
&= E_t\Big[\beta^2 \pi_{t+2}^w + \kappa_w \beta(\widehat{mrs}_{t+1} - \widehat{\omega}_{t+1}) + \kappa_w(\widehat{mrs}_t - \widehat{\omega}_t)\Big] \\
&= E_t\Big[\beta^3 \pi_{t+3}^w + \kappa_w(\widehat{mrs}_{t+2} - \widehat{\omega}_{t+2}) + \kappa_w \beta(\widehat{mrs}_{t+1} - \widehat{\omega}_{t+1}) + \kappa_w(\widehat{mrs}_t - \widehat{\omega}_t)\Big] \\
&\vdots \\
&= \kappa_w E_t \sum_{i=0}^{\infty} \beta^i (\widehat{mrs}_{t+i} - \widehat{\omega}_{t+i}),
\end{aligned}$$

where $\widehat{mrs}_t$ and $\widehat{\omega}_t$ denote marginal rate of substitution (between consumption and leisure) and real wage, respectively, in terms of deviation from flexible-price equilibrium. So we can see that wage inflation is driven by $(\widehat{mrs}_t - \widehat{\omega}_t)$. For wage adjustment workers compare the real wage to the marginal rate of substitution between leisure and consumption.

(b) Carefully explain the factors generating inefficiencies in this economy.

1. First, **imperfect competition** characterizes both the goods market and the labor market in a standard NK model with sticky prices and wages. So this is a key source of inefficiency.

2. Second, **sticky prices and sticky wages** create further distortions—each leads to a dispersion of relative prices or wages, respectively. This dispersion causes households (firms) to purchase an inefficient combination of goods (labor types). Dispersion in relative prices and wages work like negative productivity disturbances in causing more hours of work to be needed to produce a given basket of consumption. Since working generates a disutility, welfare is reduced by price and/or wage dispersion.

(c) Suppose fiscal taxes and subsidies are used to eliminate the average distortions caused by imperfect competition. Can monetary policy eliminate the remaining distortion(s)? Carefully explain your answer.

The key here is the role of the **real wage**. If the output gap is to be kept at zero (i.e., so output can move with the flex-price/wage output), then the real wage will generally need to adjust to ensure labor market equilibrium in the face of shocks. If both prices and wages are sticky, then the real wage will not be able to adjust to match what it would do in the flex-price/wage equilibrium. Monetary policy can stabilize prices (or wages), but then wage inflation (price inflation) and the output gap will move in the face of shocks since the real wage doesn't jump to ensure the output gap remains at zero. With two nominal rigidities, one policy instrument isn't sufficient.

(d) Discuss the factors that influence the relative weight the policy maker should put on maintaining price inflation and wage inflation at zero, i.e., what determines the weights $\lambda_\pi$ and $\lambda_w$ in a loss function of the form

$$\frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\lambda_\pi \pi_{t+i}^2 + \lambda_x x_{t+i}^2 + \lambda_w (\pi_{t+i}^w)^2\right]$$

where $\pi_t$ is price inflation and $\pi_t^w$ is wage inflation?

**(1) The degree of rigidity of price (wages)**

As noted in part (b), sticky prices (wages) generate a dispersion of relative prices (wages) that is inefficient. The stickier prices are, the more a given volatility of price inflation generates a dispersion of relative prices and welfare loss. The same is true for wage inflation volatility. So the policy maker should focus more weight on stabilizing the stickier of prices and wages.

**(2) Demand elasticity for goods (labor types)**

For example, relative price dispersion creates a smaller distortion if household do not respond much to relative price movements—i.e., if their demand for individual goods is relative inelastic. So if wages and prices were equally sticky (say, measured by the Calvo parameter), more weight should be put on stabilizing prices if demand facing individual firms is more elastic than the demand for individual labor types.

17. Derive the no shirking condition in the basic Shapiro-Stiglitz efficiency wage model. *Explain* how the job finding rate $a$ affects the wage. *Explain* how the job separation rate $b$ affects the wage.

The value of being employed and exerting effort:

$$\rho V_E = (w - \bar{e}) - b(V_E - V_U). \tag{1}$$

The value of being employed and shirking:

$$\rho V_S = w - (b + q)(V_E - V_U). \tag{2}$$

The value of being unemployed:

$$\rho V_U = a(V_E - V_U). \tag{3}$$

$b$ is the probability of job breakup (aka. hazard rate), $q$ is the probability or detecting shirking workers, $a$ is the probability that unemployed workers find new jobs (all in term of "per unit of time").

To not let workers shirk, firm must pay enough so that $V_E \geq V_S$. Since firm minimizes cost, $V_E = V_S$.

$$(w - \bar{e}) - b(V_E - V_U) = w - (b + q)(V_E - V_U)$$

$$\iff -\bar{e} + = -q(V_E - V_U)$$

$$\iff V_E - V_U = \frac{\bar{e}}{q}.$$

From equation (1),

$$\begin{aligned}
w &= \rho V_E + \bar{e} + b(V_E - V_U) \\
&= \rho V_E + \bar{e} + b(V_E - V_U) + \rho V_U - \rho V_U \\
&= \rho V_U + \bar{e} + (b + \rho)(V_E - V_U) \\
&= \bar{e} + (a + b + \rho)(V_E - V_U) \\
&= \bar{e} + (a + b + \rho)\frac{\bar{e}}{q}.
\end{aligned}$$

As $a \uparrow$, workers have higher probability to find jobs when unemployed; then they are more likely to shirk. So the firm should pay higher wages to prevent shirking behavior. As $b \uparrow$, the hazard rate is higher and workers are easier to lose job, so the value of employment decreases relative to the value of unemployment. Therefore, the firm needs to compensate more to prevent shirking.

18. Consider a simple DMP model that characterizes steady state unemployment, labor market tightness and the real wage:

$$u = \frac{s}{s + m_0\theta^a}$$

$$m_0\theta^{a-1}(z - w) = \kappa$$

$$w = (1 - b)R + bz$$

where $s$ is the exogenous separation rate, the matching function is $m_0 u^{1-a} v^a$, $z$ is the worker's productivity, $\kappa$ is the job posting cost, $R$ is the worker's outside opportunity, and $b$ is the worker's share of the match surplus.

(a) Explain each of these three equations.

(1) **Beveridge curve**: An equation describing the change in the unemployment rate as the difference between the inflows into unemployment and the outflows from unemployment. The steady state version of this yields the Beveridge curve, a negative relationship between vacancies and unemployment ($u$).

(2) **A theory of job/vacancy creation**: (Basically, "cost of posing vacancy = expected discounted gain from job filling"). Assuming free entry, firms will post vacancies as long as the expected return from doing so (the value of filling a job times the probability of filling it) is positive. In equilibrium, the value of a vacancy is driven to zero (i.e., $E[V] = 0$ at SS). The value of a filled job depends on the value of the output produced by a worker-firm match net of the wage the firm has to pay the worker.

(3) **A bargaining model of wages**: ($w = R + b(z - R)$: "wage = outside opportunity + worker's share in joint surplus"). A theory of wage determination. Once matched, there is a joint surplus that the firm and worker must divide. The common assumption is that Nash bargaining with fixed bargaining weights leads the surplus to be divided in fixed shares between the worker and the firm. If the worker's share is $b$, the wage is set to allocate this fraction of the joint surplus to the worker, with the firm receiving the remaining share $1 - b$.

(b) How does an increase in $R$ affect the wage, vacancies, labor market tightness, and unemployment? Explain.

$R \uparrow$ ($\Rightarrow$ value of unemployment $\uparrow \Rightarrow u \uparrow$ ) $\Rightarrow w \uparrow \Rightarrow$ vacancy($V$) $\downarrow$ (since value of job filling $\downarrow$ ) $\Rightarrow$ labor market tightness : $\theta = \frac{V}{u} \downarrow$

As the worker's outside opportunity increases, unemployment increases since the value of unemployment has increased. Accordingly, the firm needs to compensate the workers more and with increased wage, the firm has less incentive to post new jobs as the value of filling new jobs is decreased. With less job positions offered relative to unemployment, the labor market becomes less tight.

19. Each of the three core models in modern macroeconomics (RBC, new Keynesian, and DMP) encounter problems associated with their implications for employment (or unemployment) fluctuations. Describe this problem for at least two of these models. Use no more than three sentences for each model.

RBC: The RBC model misses out unemployment rate, as it assumes everyone is always employed. Labor supply is inelastic: small move in $u_t$ by a big change in $w_t$. By the RBC model, a positive productivity shock ($z_t$) increases labor supply ($N_t$), but empirical findings suggest wealth effect dominates.

NK: The model suggests inelastic labor supply.

DMP: A positive productivity shock leads to large fluctuation in wage but littler movement in unemployment. This is not consistent with US data (Shimer puzzle)

20. Discuss two models of unemployment in which $MRS_t = w_t = MPL_t$ does not hold in equilibrium.
[Check!! household markup and firm markup]
1. NK with sticky wage and sticky price: $MRS_t < w_t < MPL_t$, where the first inequality is due to household markup and the second firm markup.
2. DMP: unless the worker has all the bargaining power and takes all the joint surplus, $w_t < MPL_t$. To prevent shirking, $w_t > MRS_t$.

21. Consider a simple DMP model given by the following three equations:

$$u_t = \frac{\delta}{\delta + \theta_t \lambda(\theta_t)}; \qquad \text{Beveridge Curve} \tag{2}$$

$$\frac{c}{\lambda(\theta_t)} = \frac{q_t - w_t}{r + \delta}; \qquad \text{Job Creation Curve} \tag{3}$$

$$w_t = (1-\eta)b + \eta(1 + \theta_t c)q_t; \qquad \text{Wage Curve} \tag{4}$$

where $u$ is unemployment, $\delta$ the exogenous separation rate, $\theta = v/u$ labor market tightness, $q$ the output produced by a match, $r$ the real interest rate, $w$ the wage, $\eta$ labor's share of the surplus, $b$ is an unemployment benefit, and $cq$ is the cost of posting vacancy. Suppose $\lambda(\theta) = m_0\theta^{a-1}$ (the Hosios condition). Shimer found that in response to productivity shocks, this model implies too much wage volatility and too little unemployment volatility relative to what is observed in US data. Explain how this problem identified by Shimer could be reduced if the wage is fixed (i.e., replace (4) with $w_t = \bar{w}$ .)
Assuming sticky wage, $\bar{w} = (1-\eta)b + \eta(1 + \theta_t c)q_t$.
By the fixed wage equation, if $q_t \uparrow$, labor market tightness has to decrease (i.e., $\theta_t = v_t/u_t \downarrow$). However, facing a positive productivity shock, firms want to produce more, but since wage is fixed the only way to produce more is to post more jobs ($v \uparrow$). So, unemployment $u_t$ needs to increase a lot to outweigh the decrease in $v_t$. Therefore, we have higher volatility in $u_t$.
To understand how a positive productivity shock leads to a lower labor market tightness with sticky wages,
$q_t \uparrow \Rightarrow MPL_t \uparrow \Rightarrow N_t$ fixed ($\because\ w_t = \bar{w}$) (by substitution effect)
$q_t \uparrow \Rightarrow Y_t \uparrow \Rightarrow N_t \downarrow$ (by wealth effect)
In total, $N_t \downarrow \Rightarrow u_t \uparrow \Rightarrow \theta_t \uparrow$

22. In the Mortensen-Pissarides model, explain why equilibrium unemployment may be too high. When might it be too low?
If firms get too little share of the joint surplus, there will be too little vacancies and unemployment level is inefficiently high.
If firms get too much share of the joint surplus, there will be too many vacancies and unemployment level is inefficiently low.
By the Hosios condition, the level of unemployment is efficient when $(1-a)$ from the matching function is equal to $\eta$ in the Beveridge curve. That is, labor share in joint surplus = unemployment's share in the matching function.

23. Consider an economy in which job matches are determined by an aggregate matching function of the form

$$m(v, u) = m_0 v^{1-\alpha} u^{\alpha},$$

where $v$ is the number of job vacancies and $u$ is the number of unemployed workers searching for jobs. Assume there is an exogenous rate at which jobs (worker-firm matches) break-up. Firms face a constant (per period) cost of $c$ if they post a vacancy. Each filled job produces output of $y$ per period. Let $w$ denote the worker's wage. The real interest rate is $r$.

(a) Derive the Beveridge curve between $u$ and $\theta = v/u$ along which $u$ is constant. Explain why it implies a negative relationship between $\theta$ and $u$.

$$\frac{m(v,u)}{v} = m_0\left(\frac{v}{u}\right)^{-\alpha} = m_0\theta^{-\alpha} \quad \text{(job filling rate)}$$
$$\frac{m(v,u)}{u} = m_0\left(\frac{v}{u}\right)^{1-\alpha} = m_0\theta^{1-\alpha} \quad \text{(job finding rate)}$$

The change in unemployment is inflow (employed workers being "separated" from firm) minus outflow (the unemployed being "matched" with firm):

$$\begin{aligned}\dot{u} &= e\cdot\delta - m(v,u)\\ &= e\cdot\delta - u\cdot\frac{m(v,u)}{u}\\ &= e\cdot\delta - u\cdot m_0\theta^{1-\alpha}\end{aligned}$$

At SS, $\dot{u} = 0$,

$$e\cdot\delta = u\cdot m_0\theta^{1-\alpha}$$

Since $1 = e + u$,

$$(1-u)\cdot\delta = u\cdot m_0\theta^{1-\alpha} \iff u = \frac{\delta}{\delta + m_0\theta^{1-\alpha}}$$

An increase in $\theta_t$ means the labor market is tighter. Since there are more vacancies relative to unemployment, it is easier for workers to find job. And this leads to a lower level of unemployment.

(b) What is the equilibrium value of an unfilled vacancy? Why? Write down the value equation for a vacancy and derive an equilibrium condition for the value of a filled job ($J$) in terms of $\theta$ and $c$.
The value equation for a vacancy is

$$\begin{aligned}rV &= -c + \frac{m(v,u)}{v}\cdot J + (1 - \frac{m(v,u)}{v})\cdot V\\ &= -c + \theta^{-\alpha}\cdot J + (1-\theta^{-\alpha})\cdot V\\ &= -c + m_0\theta^{-\alpha}(J - V).\end{aligned}$$

Assuming free entry in vacancy posting, the value of unfilled vacancy is zero at equilibrium, so

$$J = \frac{c}{m_0\theta^{-\alpha}}.$$

That is, the value of a filled job must compensate the firm for the cost of filling the job. As the job filling rate increases (i.e., vacancies are filled more quickly), the expected cost of filling a job falls. Expressed differently, when jobs become easier to fill, firms post more vacancies and employs more. This reduces the marginal value of a filled job.

(c) What is the equilibrium value of a filled job to the firm? Express this value in terms of $y - w, r$,

and $\delta$.
The value equation for a filled job is

$$rJ = y - w - \delta(J - V).$$

At equilibrium, $V = 0$, so

$$rJ = y - w - \delta J$$
$$J = \frac{y - w}{r + \delta}$$

(d) Use the expression for $J$ obtained in (c) in the condition you derived in (b) to obtain a relationship between $w$ and $\theta$. Explain intuitively what this equilibrium condition implies about the relationship between wages and labor market tightness.
From parts (b) and (c),

$$\frac{c}{m_0\theta^{-\alpha}} = \frac{y - w}{r + \delta}$$
$$w = y - \frac{c(r + \delta)}{m_0\theta^{-\alpha}}$$

As labor tightness ($\theta$) increases, wage ($w$) decreases. As the labor market become more tight, more jobs are posted relative to unemployment, the value of jobs decrease and so does wage.

24. Consider a standard new Keynesian model with sticky prices and wages; both adjust according to a simple Calvo model but with different degrees of stickiness.

(a) What are the driving variables for price inflation and wage inflation?

(b) Carefully explain the factors generating inefficiencies in this economy.

(c) Suppose fiscal taxes and subsidies are used to eliminate the average distortions caused by imperfect competition. Can monetary policy eliminate the remaining distortion(s)? Carefully explain your answer.

(d) Discuss the factors that influence the relative weight the policy maker should put on maintaining price inflation and wage inflation at zero, i.e., what determines the weights $\lambda_\pi$ and $\lambda_w$ in a loss function of the form

$$\frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\lambda_\pi \pi_{t+i}^2 + \lambda_x x_{t+i}^2 + \lambda_w (\pi_{t+i}^w)^2\right]$$

where $\pi_t$ is price inflation and $\pi_t^w$ is wage inflation?
See Problem 16 for parts (a) to (d).

(e) In the absence of inflation shocks, is it possible for the central bank to maintain zero inflation and a zero output gap in the face of the productivity shocks? Explain.

25. Consider a new Keynesian model of a small open economy (SOE). Suppose it experiences a negative domestic productivity shock.

(a) How are the output gap and cpi inflation affected if the central bank tries to stabilize domestic price inflation? What happens to the terms of trade and the nominal exchange rate. Explain.
If the CB want to stabilize price inflation, it wants to keep the output gap at zero.

$z_t \downarrow \Rightarrow y_t^f \downarrow \Rightarrow$ CB's contractionary policy (to keep $x_t = 0$) $\Rightarrow y_t \downarrow \Rightarrow i_t \uparrow$ (value of money $\uparrow$) $\Rightarrow$ domestic currency appreciation (by UIP) $\Rightarrow e_t \downarrow \Rightarrow P_t^f (= e_t \cdot P_t^{*f}) \downarrow \Rightarrow S_t (= \frac{P_t^f}{P_t^h}) \downarrow$ and $P_t \downarrow$
$\Rightarrow$ A negative domestic productivity shock leads to a decrease in domestic flex-price output (i.e., $y_t^f \downarrow$). Since the CB wants to stabilize domestic price inflation, it should keep the output gap ($x_t$) at zero (by the PC). So the output ($y_t$) should decrease in response to $y_t^f \downarrow$. If the CB carries out a contractionary policy (to lower $y_t$), nominal interest rate goes up as people want to invest in and hold more domestic currency. Then, domestic currency appreciates until agents depreciation (by the uncovered interest rate parity), and this leads to a decrease in exchange rate, which is followed by a decrease in terms of trade. Although $P_t^h$ is stabilized by the CB, since $P_t^f$ decreases, the CPI inflation decreases.

(b) How are the output gap and cpi inflation affected if the central bank tries to stabilize the nominal exchange rate? What happens to the terms of trade and domestic price inflation. Explain.
If the CB want to stabilize nominal exchange rate, it wants to stabilize $i_t$, by the UIP

$$i_t = i_t^* + \mathrm{E}_t(e_{t+1} - e_t).$$

$z_t \downarrow \Rightarrow y_t^f \downarrow \Rightarrow x_t \uparrow \Rightarrow \pi_t^h \uparrow \Rightarrow P_t^h \uparrow \Rightarrow S_t \downarrow$ and $P_t \downarrow (SR) \uparrow (LR)$
Since the CB wants to stabilize $i_t$, it will have no contractionary policy to move $y_t$. So, $x_t$ increases and by the PC the domestic inflation increases. This leads to a decrease in terms of trade. Even though domestic price inflation increases ($\pi_t^h \uparrow$), CPI inflation will decrease in the short run ($\pi_t \downarrow$) due to a fall in TOT ($s_t \downarrow$) but will increase in the long run ($\pi_{t+1} \uparrow$).

$$\pi_t = \pi_t^h + \gamma(s_t - s_{t-1})$$

$$\pi_{t+1} = \pi_{t+1}^h + \gamma(s_{t+1} - s_t)$$

**Part B: no equations, explain in only words**

1. Suppose the economy experiences a negative aggregate demand shock that pushes the nominal interest rate to zero. How would increased pessimism about how long the economy will be at the zero lower bound will affect the current equilibrium output gap and inflation? Explain.
There are several channels at work here.
(1) Current demand:
Current demand depends on expected future demand, so if households become more pessimistic about future income, current demand will fall and this will reduce current output. The fall in current demand and output has a direct effect in reducing current inflation.
(2) Expected future inflation:
A more pessimistic view of future output also lowers expected future inflation. This increases the current real interest rate (given the nominal interest rate can't adjust). So the rise in the real interest rate further depresses current aggregate demand and thus output gap. The fall in expected future inflation also has a direct effect in reducing current inflation.

2. In the Diamond-Dybvig model of bank runs, why are banks able to create liquidity? How does this make them vulnerable to runs?
Banks purchase illiquid asset and issue liquid deposit liabilities. Banks may have liquidity problems when if too many depositors want to withdraw their money at $T = 1$, then banks don't have enough money to

pay the rest of depositors at $T = 2$ with promised returns. If depositors expected many withdrawals at $T = 1$, all the investors would want to withdraw at $T = 1$ (i.e., bank run). However, the banks cannot repay all the investors because the return of banks' assets is lower than the promised return of deposit at $T = 1$.

3. Each of the three core models in modern macroeconomics (RBC, new Keynesian, and DMP) encounter problems associated with their implications for employment (or unemployment) fluctuations. Describe this problem for at least two of these models. Use no more than three sentences for each model.
   See Problem 19.

4. Suppose a sticky-price new Keynesian model is given by

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

   If the central bank wants to ensure $x_t = \pi_t = 0$, what problems might arise if the central bank sets its interest rate instrument according to the rule $i_t = r_t^n$?
   See Problem 6.