Consider an RBC model where consumption exhibits habit persistence. For simplicity, we assume there is no capital in the economy. The household's problem is to choose c_t, h_t to maximize the discounted-sum of utility:

$$\max_{c_t, h_t} E \sum_{t=0}^{\infty} \beta^t u(\cdot) \qquad 0 < \beta < 1$$

$$s.t. \quad c_t + i_t \leq w_t h_t$$

The production function is given by $Y_t = z_t H_t$. The technology shock follows a standard AR(1) process ($\ln z_t = \rho \ln z_{t-1} + \epsilon_t$) and the resource constraint is given by $C_t = Y_t$.

1. Assume the utility function is given by

$$u(c_t, c_{t-1}, h_t) = \ln(c_t - bc_{t-1}) - \frac{h_t^{1+\eta}}{1+\eta}$$

   where $b > 0$ and $\eta > 0$ are parameters. Thus, households exhibit consumption habit with respect to their own past consumption levels.

   (a) Define the recursive competitive equilibrium, including the household's problem and the firm's problem.

   (b) Derive the equilibrium conditions of this economy.

2. Assume the utility function is given by

$$u(c_t, C_{t-1}, h_t) = \ln(c_t - bC_{t-1}) - \frac{h_t^{1+\eta}}{1+\eta}$$

   where $b > 0$ and $\eta > 0$ are parameters. C_{t-1} is the period $t-1$ *aggregate* consumption. Thus, households exhibit consumption habit with respect to the past *aggregate* consumption level. (You can think of this preference as representing "keeping up with the peers" effect.)

   (a) Define the recursive competitive equilibrium, including the household's problem and the firm's problem.

   (b) Derive the equilibrium conditions of this economy.

   (c) Suppose the government wishes to improve welfare by imposing a labor income tax τ_t to households. Suppose the government balances budget period-by-period and the tax revenue is rebated back to the households. What would the optimal labor income tax look like over the business cycle? Briefly explain.