Consider an RBC model with investment-specific technology shocks (i.e., investment shocks). The household's problem is to choose c_t, h_t, i_t to maximize the discounted-sum of utility:

$$\max_{c_t,h_t,i_t} E\sum_{t=0}^{\infty}\beta^t u(c_t,h_t), \qquad 0<\beta<1$$

$$s.t. \quad c_t + i_t \le w_t h_t + r_t k_{t-1}$$

$$k_t = (1-\delta)k_{t-1} + \mu_t i_t$$

where we specify

$$u(c_t,h_t) = \ln c_t - \varphi\frac{h_t^{1+\eta}}{1+\eta}$$

where φ and η are parameters. The investment-specific technology shock follows $\ln\mu_t = \rho\ln\mu_{t-1} + \epsilon_{\mu,t}$. The production side is the same as in the baseline RBC model (Perfectly competitive firms with production function $Y_t = z_t K_{t-1}^{\alpha} H_t^{1-\alpha}$ with neutral technology (TFP) shock $\ln z_t = \rho\ln z_{t-1} + \epsilon_{z,t}$, etc.).

1. Define the recursive competitive equilibrium, including the household's problem and the firm's problem.

2. State the sequential social planner's problem for this economy.

3. Derive the equilibrium conditions of this economy.

4. In response to a positive investment shock (an increase in μ_t), investment increases but consumption declines. Explain briefly why this happens.

5. Suppose the government wishes to completely offset the effects of investment shocks by changing the tax rates on capital rental income. Derive the expression for the optimal capital income tax.