Solve **both** part 1 and 2.

1. One of the problems in calibrated RBC models is that investment is too volatile compared to the data. Thus, we consider an RBC model with capital adjustment cost. The household's problem is to choose c_t, h_t, i_t to maximize the discounted-sum of utility:

$$\max_{c_t,h_t,i_t} E\sum_{t=0}^{\infty}\beta^t u(c_t,h_t), \qquad 0<\beta<1$$

$$s.t. \quad c_t + i_t \leq w_t h_t + r_t k_{t-1}$$

$$k_t = (1-\delta - S(i_t/k_{t-1}))k_{t-1} + i_t$$

where we specify

$$u(c_t,h_t) = \ln c_t - \varphi\frac{h_t^{1+\eta}}{1+\eta}$$

where φ and η are parameters. $S(i_t/k_{t-1})$ is the adjustment cost. We assume

$$S\left(\frac{i_t}{k_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t}{k_{t-1}} - \delta\right)^2$$

where $\kappa > 0$ is a parameter. Note that when $i_t/k_{t-1} = \delta$, there is no penalty for adjusting the stock of capital. The production side is the same as in the baseline RBC model (Perfectly competitive firms with production function $Y_t = z_t K_{t-1}^{\alpha} H_t^{1-\alpha}$ with neutral technology (TFP) shock $\ln z_t = \rho \ln z_{t-1} + \epsilon_{z,t}$, etc.).

   (a) Define the recursive competitive equilibrium, including the household's problem and the firm's problem.

   (b) Derive the equilibrium conditions for this economy.

2. Recall the IS and NKPC blocks of the New Keynesian model:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - E_t\pi_{t+1}) + u_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t$$

where u_t and e_t are shocks. The central bank's objective is to minimize

$$L_t = \left(\frac{1}{2}\right)E_t\sum_{i=0}^{\infty}\beta^i(\pi_{t+i}^2 + \lambda x_{t+i}^2)$$

   (a) Derive the first order condition(s) for the optimal policy under commitment (assume timeless perspective approach).

(b) Derive the first order condition(s) for the optimal policy under discretion.