205B 2016

Midterm: brief solution
(not complete)

1.(a) HH's problem

$$\begin{cases} \max_{c_t, h_t, i_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\ln c_t - \varphi_t \frac{h_t^{1+\eta}}{1+\eta} \right] \\ \text{s.t. } c_t + i_t \le w_t h_t + r_t k_{t-1} \\ \quad k_t = (1-\delta) k_{t-1} + i_t \end{cases}$$

Firm's problem

(see slide)

SME is similar to the slide except additional exog. state variable φ_t.

(b) HH's problem

$V(k, K, z, \varphi)$

$$= \max_{c,h,i} \left[u(c,h) + \beta \mathbb{E} V(k', K', z', \varphi') \right]$$

s.t. $c + i \le wh + rk$

$k' = (1-\delta)k + i$.

Firm's problem (see slide)

Again, RCE is similar to the one in slide except there is one additional exog. state φ.

(c)

$$\max_{C_t, H_t, I_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \varphi_t \frac{H_t^{1+\eta}}{1+\eta} \right]$$

$$\text{s.t. } C_t + I_t \le z_t K_{t-1}^{\alpha} H_t^{1-\alpha}$$

$$K_t = (1-\delta) K_{t-1} + I_t.$$

(d)

$$\frac{1}{C_t} \cdot w_t = \varphi_t \frac{H_t^{1+\eta}}{1+\eta}.$$

(For other conditions, see slide)

(e) $\varphi_t \uparrow$ (more disutil. from working).

$\rightarrow H \downarrow$, so $Y \downarrow$,

income down, so $C \downarrow$.

(f) Add additional state φ_ℓ, where n_φ is size of grid,

$$V(K, z_i, \varphi_\ell)$$

$$= \max_{K'} \left[\ln C - \varphi_\ell \frac{H^{1+\eta}}{1+\eta} + \beta \sum_{j=1}^{n_z} \sum_{m=1}^{n_\varphi} V(K', z_j, \varphi_m) \right].$$

2(a)

$$V(K_M, K_H, I_{M,-1}, Z_M, Z_H)$$

$$= \max_{\left\{\begin{array}{l} K_M', K_H', I_M, \\ Z_M', Z_H' \end{array}\right\}} \left[u(c, \ell) + \beta \mathbb{E} V(K_M', K_H', I_M, Z_M, Z_H) \right]$$

S.T. Resource constraints & Capital accumulations.

(b)

$$\mathcal{L} = \mathbb{E} \sum \beta^t \Big[u(c_t, \ell_t)$$

$$+ \lambda_t \{ f(\quad) - c_{M,t} - i_{M,t}$$

$$- k_{H,t} + (1-\delta) k_{H,t-1} \}$$

$$+ \mu_t \{ (1-\delta) k_{M,t-1} + \tfrac{1}{2} i_{M,t} + \tfrac{1}{2} i_{M,t-1} - k_{M,t} \}$$

$$+ \varphi_t \{ g(\quad) - c_{H,t} \} \Big]$$

FONC w.r.t. $\lambda_{m,t}$:

$$\lambda_t = \frac{1}{2}\mu_t + \frac{1}{2}\beta \mathbb{E}_t \mu_{t+1}$$

Other eq conditions as in the lecture.

(c) Can weaken the negative correlation between business & home investment by smoothing market investment.