Midterm
Econ 205B, Winter 2016

- *You have 60 minutes to complete the exam. The maximum points possible is 50.*
- *Be concise. Long answers with redundant statements, even if they contain correct answers, will likely be penalized.*

1. Consider an RBC model with an intratemporal preference shock. The household's problem is to choose c_t, h_t, i_t to maximize the discounted-sum of utility:

$$\max_{c_t,h_t,i_t} E\sum_{t=0}^{\infty}\beta^t u(c_t,h_t), \qquad 0<\beta<1$$
$$s.t. \quad c_t + i_t \leq w_t h_t + r_t k_{t-1}$$
$$k_t = (1-\delta)k_{t-1} + i_t$$

where we specify

$$u(c_t,h_t) = \ln c_t - \varphi_t \frac{h_t^{1+\eta}}{1+\eta},$$

where η is a parameter. φ_t is an intratemporal preference shock that follows an AR(1) process:

$$\ln \varphi_t = \rho_\varphi \ln \varphi_{t-1} + \epsilon_{\varphi,t}$$

The production side is the same as in the baseline RBC model (Perfectly competitive firms with production function $Y_t = z_t K_{t-1}^{\alpha} H_t^{1-\alpha}$ with aggregate technology shock $\ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t}$, etc.).

(a) (5 points) Define the sequential market equilibrium, including the household's problem and the firm's problem.

(b) (5 points) Define the recursive competitive equilibrium, including the household's problem and the firm's problem.

(c) (5 points) State the sequential social planner's problem for this economy.

(d) (5 points) Derive the equilibrium conditions of this economy.

(e) (5 points) What happens to consumption, hours, and output when there is a positive shock to φ (an increase in φ)? Briefly explain.

(f) (5 points) Suppose you already have a code for the value function iteration that solves a standard RBC model (that is, a model without the intratemporal preference shock). How would you modify the code to solve the model with the intratemporal preference shock? Briefly explain.

2. Consider an RBC model with home production and time-to-build investment technology for market capital.[1] The instantaneous utility is given by

$$u(c, l) = \log(c) + A \log(l)$$

where

$$c_t = [ac_{M,t}^e + (1-a)c_{H,t}^e]^{1/e}$$
$$l_t = 1 - h_{M,t} - h_{H,t}$$

The market and home production technologies are given by

$$f(z_{M,t}, k_{M,t-1}, h_{M,t}) = \exp(z_{M,t}) k_{M,t-1}^{\theta} h_{M,t}^{1-\theta}$$
$$g(z_{H,t}, k_{H,t-1}, h_{H,t}) = \exp(z_{H,t}) k_{H,t-1}^{\eta} h_{H,t}^{1-\eta}$$

where

$$z_{M,t+1} = \rho z_{M,t} + \epsilon_{M,t},$$
$$z_{H,t+1} = \rho z_{H,t} + \epsilon_{H,t}.$$

Capital accumulation equations are

$$k_{M,t} = (1-\delta)k_{M,t-1} + \frac{1}{2} i_{M,t} + \frac{1}{2} i_{M,t-1}$$
$$k_{H,t} = (1-\delta)k_{H,t-1} + i_{H,t}$$

Resource constraints are

$$c_{M,t} + i_{M,t} + i_{H,t} = f(z_{M,t}, k_{M,t-1}, h_{M,t})$$
$$c_{H,t} = g(z_{H,t}, k_{H,t-1}, h_{H,t})$$

(a) (5 points) State the recursive social planner's problem for this economy.

[1] As we see below, when households undertake 1 unit of market investment at period t, only 1/2 unit turns into market capital in the same period and the rest turns into market capital at the end of period t + 1.

(b) (10 points) Derive the equilibrium conditions of this economy.

(c) (5 points) What is the advantage, in terms of matching data, of introducing the time-to-build assumption for market capital accumulation? Briefly explain.