Midterm 2015

Brief Answers

1.(a) Exactly the same def. as in the baseline RBC model in the slide.

(Except. util func. is

$$u = \ln(c_t - \underline{c}) - \varphi \frac{h_t^{1+\eta}}{1+\eta}$$
)

(b) Exactly the same problem as in the baseline RBC model in the slide.

(Again, except for the util. func.)

HH's problem

1.(c) $\mathcal{L} = \mathbb{E} \sum \beta^t \Big[\ln(c_t - \underline{c}) - \varphi \frac{h_t^{1+\eta}}{1+\eta}$

$+ \lambda_t \{ w_t h_t + r_t k_{t-1} - c_t$

$- k_t + (1-\delta) k_{t-1} \} \Big]$

FONC. c_t

$$\lambda_t = \frac{1}{c_t - \underline{c}}$$

FONC. h_t

$$\lambda_t w_t = \varphi h_t^{\eta}$$

FONC k_t

$$\lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} (r_{t+1} + 1 - \delta) \right].$$

and combine them with firm's FONC and market clearing conditions.

1.(d) Similar to RCE for the baseline RBC model, but now the exogenous state variables are z and $\underline{c}$.

(e) When $\underline{c}$ increases, marginal util. of consumption (λ_t) increase for a given consumption level.

→ Consumption increases.

2(a) $\xi_\tau = -g_\tau$

<u>Note</u>

(b) The production function should have been

$$Y_\tau = Z_\tau (K_{\tau-1} + \underline{\underline{g_\tau}})^\alpha H_\tau^{1-\alpha} \quad (1)$$

instead of

$$Y_\tau = Z_\tau (K_{\tau-1} + g_{\tau-1})^\alpha H_\tau^{1-\alpha} \quad (2)$$

The solution that follows uses the corrected version of production func (1).

Solutions under the original production func. (2) are given full marks if they are correct under the assumption (2).

2. (b) HH's problem

$$V(k, K, z, g)$$

$$= \max_{c, h, i} \left[u(c, h) + \beta \mathbb{E} \{ V(k', K', z', g) \} \right]$$

$$\text{s.t. } c + i \leq w h + r k + \xi$$

Firm's problem

$$\max_{K^d, H^d} \left[z (K^d + g)^\alpha (H^d)^{1-\alpha} - r K^d - w H^d \right]$$

RCE is

(i) HH's value $V(k, K, z, g)$,

policy $c(k, K, z, g)$, $h(k, K, z, g)$, $i(k, K, z, g)$

(ii) Firm's policy $H^d(K, z, g)$, $K^d(K, z, g)$.

(iii) prices $w(K, z, g)$, $r(K, z, g)$.

(iv) LoM of capital $K' = m(K, z, g)$.

such that

- Given prices, LoM of capital, gov. transfer func, HH's policy func, solve the HH's Bellman equation.
- Given prices, firm's policy function solve the firm's problem.
- The GBC is satisfied.
- Markets clean

$$\begin{cases} - h = H = H^d \\ - k = K = K^d. \\ - c + i + g = z(K+g)^{\alpha} H^{1-\alpha} \end{cases}$$

- Consistency

$$m(K, z, g) = (1-\delta)K + i(K, K, z, g)$$

2.(c) Household's FONCs are usual.

Firm's FONCs.

$$\begin{cases} w_t = (1-\alpha)\dfrac{Y_t}{H_t}. \\[2ex] r_t = \alpha \dfrac{Y_t}{K_{t-1}+g_t}. \end{cases}$$

Resource constraint

$$C_t + I_t + g_t = Y_t.$$

(d)

$$g \uparrow \rightarrow \begin{cases} Y \uparrow \text{ (from direct increase in input), and also labor.} \\ H \uparrow \text{ due to increase in labor demand and income effect of labor supply} \end{cases}$$

Investment response ambiguous, depends on the magnitude of increase in income vs. response of consumption.