205B Midterm W. 2014

Brief answers.

1. (a) SME is

(i) HH's policy func.

$c(h_{t-1}, k_{t-1}, H_{t-1}, K_{t-1}, z)$,

$h($ —— $)$, $i($ —— $)$,

(ii) Firm policy func.

$H^d(H_{t-1}, K_{t-1}, z_t)$, $K^d($ —— $)$.

(iii) Prices. $w(H_{t-1}, K_{t-1}, z_t)$, $r^d($ —— $)$

(iv) LOM for capital.

$K_t = g(H_{t-1}, K_{t-1}, z_t)$.

- HH's problem

$$\begin{cases} \max\limits_{c_t, h_t, i_t} E \sum \beta^t u(c_t, h_t, h_{t-1}). \\ \text{s.t.} \quad c_t + i_t \leq w_t h_t + r_t k_{t-1} \\ \qquad k_t = (1-\delta) k_{t-1} + i_t. \end{cases}$$

- Firm's problem.

$$\max_{K_t^d, H_t^d} [\text{ —— }].$$

- Market clearing
  - ______ (Labor)
  - ______ (Capital)
  - ______ (Goods)

- Consistency condition.

$g(H_{t-1}, K_{t-1}, z_t)$

$= (1-\delta)K_{t-1} + i(H_{t-1}, K_{t-1}, H_{t-1}, K_{t-1}, z_t)$

(b) RCE is

(i) HH's value func $V(h_{-1}, k, H_{-1}, K, z)$

policy func. $c(\text{——})$, $h(\text{——})$, $i(\text{——})$

(ii) Firm's policy func.

$H^d(H_{-1}, K, z)$, $K^d(H_{-1}, K, z)$

(iii) Prices $w(H_{-1}, K, z)$, $r(\text{——})$

(iv) LOM for capital $K' = g(H_{-1}, K, z)$.

- HH's problem.

$$
\begin{cases}
V(h_{-1}, k, H_{-1}, K, z) \\
\quad = \max\limits_{c, h, i} \left[u(c, h, h_{-1}) + \beta \mathbb{E} V(h, k', H, K', z') \right] \\
\quad \text{s.t.} \quad c + i \le w h + r k, \\
\qquad\qquad k' = (1-\delta) k + i.
\end{cases}
$$

- Firm's problem.

$$\max_{K^d, H^d} \left[\quad \rule{4cm}{0.4pt} \quad \right]$$

- Market clearing.

  - ____________
  - ____________
  - ____________

- Consistency condition.

$$g(H_{-1}, K, z) = (1-\delta) K + i(H_{-1}, K, H_{-1}, K, z).$$

(d).

$$\begin{cases} V(H_{-1}, K, z) \\ \quad = \max_{C, H, I} \left[u(C, H_{-1}, H) + \beta \mathbb{E} V(H, K', z') \right]. \\ \text{s.t.} \quad C + I \leq zF(K, H). \\ \qquad K' = (1-\delta)K + I. \end{cases}$$

(e)

$$\mathcal{L} = \mathbb{E} \sum \beta^t \Big\{ \ln c_t - \varphi \frac{(h_t - b h_{t-1})^{1+\eta}}{1+\eta}$$

$$+ \lambda_t \left[w_t h_t + r_t k_{t-1} - c_t - k_t + (1-\delta) k_{t-1} \right] \Big\}$$

- FONC for c_t.

$$\lambda_t = \frac{1}{c_t}$$

- FONC for h_t.

$$\lambda_t w_t - \varphi (h_t - b h_{t-1})^\eta$$
$$+ \varphi \beta b \mathbb{E}_t (h_{t+1} - b h_t)^\eta = 0.$$

- FONC for k_t,

$$\lambda_t = \beta E_t[\lambda_{t+1}(r_{t+1} + 1 - \delta)]$$

Also solve for firm's problem and impose market clearing conditions.

(e). Since there is a "habit-persistence" in ind. hours worked, hours become less volatile and more auto-correlated.

---

2. (a) RCE is

(i) HH's value func. $V(k, H_{-1}, K, z)$, policy func. $c(\text{——})$, $h(\text{——})$, $i(\text{——})$

(ii) Firm's policy func.

$H^d(H_{-1}, K, z)$, $K^d(\text{——})$.

(iii) Prices $w(H_{-1}, K, z)$, $r(\text{——})$

(iv) LOM for capital $K' = g(H_{-1}, K, z)$.

- HH's problem.

$$\begin{cases} V(k, H_{-1}, K, z) \\ = \max_{c, h, i} \left[u(c, h, h_{-1}) + \beta E V(k', H, K', z') \right] \\ \text{s.t. } c + i \leq w h + r k \\ \quad k' = (1-\delta) k + i \end{cases}$$

- Firm's problem.

$$\max_{K^d, H^d} [\longrightarrow]$$

- Market clearing.

  - ________
  - ________
  - ________

- Consistency.

$$g(H_{-1}, K, z) = (1-\delta) K + i(K, H_{-1}, K, z).$$

2.(b).

$$\mathcal{L} = \mathbb{E}\sum \beta^{\tau}\Big\{\ln c_{\tau} - \varphi\frac{(h_{\tau} - bH_{\tau-1})^{1+\eta}}{1+\eta}$$

$$+\lambda_{\tau}[w_{\tau}h_{\tau} + r_{\tau}k_{\tau-1} - c_{\tau} - k_{\tau} + (1-\delta)k_{\tau-1}]\Big\}$$

The only difference from 1(d) would be the labor supply cond;

FONC for h_{τ}

$$\lambda_{\tau}w_{\tau} - \varphi(h_{\tau} - bH_{\tau-1})^{\eta} = 0.$$

Imposing market clearing cond,

$$\lambda_{\tau}w_{\tau} = \varphi(H_{\tau} - bH_{\tau-1})^{\eta}$$

3. (a) $\xi_\tau = -\tau_\tau w_\tau H_\tau$.

(b).

$$\max_{K_\tau^d, H_\tau^d} \left[(K_\tau^d)^\alpha (H_\tau^d)^{1-\alpha} - r_\tau K_\tau^d - (1-\tau_\tau) w_\tau H_\tau^d \right].$$

<u>FONC w.r.t. K_τ^d</u>.

$$\alpha (K_\tau^d)^{\alpha-1} (H_\tau^d)^{1-\alpha} = r_\tau.$$

<u>FONC w.r.t. H_τ^d</u>

$$(1-\alpha)(K_\tau^d)^\alpha (H_\tau^d)^{-\alpha} = (1-\tau_\tau) w_\tau.$$

(c).

- $\lambda_\tau = \frac{1}{C_\tau}$.
- $\lambda_\tau w_\tau = \varphi H_\tau^\eta$.
- $\lambda_\tau = \beta \mathbb{E}_\tau [\lambda_{\tau+1} (r_{\tau+1} + 1 - \delta)]$.
- $r_\tau = \alpha K_{\tau-1}^{\alpha-1} H_\tau^{1-\alpha}$
- $(1-\tau_\tau) w_\tau = (1-\alpha) K_{\tau-1}^\alpha H_\tau^{-\alpha}$.

- $K_t = (1-\delta) K_{t-1} + I_t$,

- $C_t + I_t = K_{t-1}^{\alpha} H_t^{1-\alpha} = Y_t$.

(e) <u>HH's FONC for h_t.</u>

$\rightarrow \lambda_t w_t = \varphi_t H_t^{\eta}$.

Plug in the expression for w_t:

$(1-\tau_t) w_t = (1-\alpha) K_{t-1}^{\alpha} H_t^{-\alpha}$

$$\rightarrow \lambda_t \frac{(1-\alpha) K_{t-1}^{\alpha} H_t^{-\alpha}}{(1-\tau_t)} = \varphi_t H_t^{\eta}$$

so the gov. want to set

$$\varphi_t (1-\tau_t) = 1$$

or $\underline{\tau_t = 1 - \frac{1}{\varphi_t}}$ //

Note in 3(c):

The HH's lagrangian is

$$\mathcal{L} = \mathbb{E}\sum \beta^t \Big\{ \ln c_t - \varphi \frac{h_t^{1+\eta}}{1+\eta}$$

$$+ \lambda_t \Big[r_t k_{t-1} + w_t h_t \underbrace{- \tau_t w_t \underline{\underline{H_t}}}_{= \xi_t} - c_t - i_t \Big] \Big\},$$

Since ξ_t is not affected by individual h_t, it does not affect FONC for h_t.