Midterm
Econ 205B, Winter 2014

- *You have 90 minutes to complete the exam. The maximum points possible is 100.*
- *No question can be asked during the exam. If you are unsure about the question, state clearly your interpretation and answer appropriately.*
- *Be concise. Long answers with redundant statements, even if they contain correct answers, will likely be heavily penalized.*

1. Consider an RBC model with non-time-separable labor preference. The household's problem is to choose c_t, h_t, i_t to maximize the discounted-sum of utility:

$$\max_{c_t,h_t,i_t} E\sum_{t=0}^{\infty}\beta^t u(c_t,h_t,h_{t-1}), \qquad 0<\beta<1$$
$$s.t. \quad c_t + i_t \le w_t h_t + r_t k_{t-1}$$
$$k_t = (1-\delta)k_{t-1} + i_t$$

where we specify

$$u(c_t,h_t,h_{t-1}) = \ln c_t - \varphi\frac{(h_t - bh_{t-1})^{1+\eta}}{1+\eta}, \qquad 0<b<1$$

where φ, b, and η are parameters. Thus the household's individual labor dis-utility depends on the past individual hours worked. The production side is the same as in the baseline RBC model (Perfectly competitive firms with production function $Y_t = z_t K_{t-1}^{\alpha} H_t^{1-\alpha}$ with aggregate technology shock $\ln z_t = \rho \ln z_{t-1} + \epsilon_t$, etc.).

   (a) (10 points) Define the sequential market equilibrium, including the household's problem and the firm's problem.

   (b) (10 points) Define the recursive competitive equilibrium, including the household's problem and the firm's problem.

   (c) (10 points) State the recursive social planner's problem for this economy.

   (d) (10 points) Derive the equilibrium conditions of this economy.

   (e) (5 points) Explain the differences in the behavior of hours worked (standard deviation, autocorrelation etc.) in this model to the baseline RBC model (where we have $b = 0$).

2. Now let's specify the household's utility function as follows:

$$u(c_t, h_t, H_{t-1}) = \ln c_t - \varphi \frac{(h_t - bH_{t-1})^{1+\eta}}{1+\eta}, \qquad 0 < b < 1$$

so that the household's individual labor dis-utility depends on the past *aggregate* hours worked. The rest of the model is identical to the previous question.

   (a) (10 points) Define the recursive competitive equilibrium, including the household's problem and the firm's problem.

   (b) (10 points) Derive the equilibrium conditions of this economy.

3. We want to consider the effects of a hiring subsidy on business cycles. In particular, perfectly competitive firms with a production function $Y_t = K_{t-1}^{\alpha} H_t^{1-\alpha}$ pay $(1-\tau_t)w_t H_t^d$ when they hire H_t^d units of labor. (They pay $r_t K_t^d$ when they rent K_t^d units of capital.) The hiring subsidy rate τ_t follows an exogenous AR(1) process

$$\tau_{t+1} = (I - \rho_\tau)\bar{\tau} + \rho_\tau \tau_t + \epsilon_{\tau,t+1}, \qquad \epsilon_{\tau,t+1} \sim N(0, \sigma_\tau^2).$$

with $0 < \bar{\tau} < 1$.

Household maximizes utility

$$E \sum_{t=0}^{\infty} \beta^t \left[\ln c_t - \varphi \frac{h_t^{1+\eta}}{1+\eta} \right]$$

subject to

$$c_t + i_t \leq r_t k_{t-1} + w_t h_t + \xi_t$$

where ξ_t is the government transfer.

For simplicity, assume there is no government consumption (i.e., $g_t = 0$ for all t).

   (a) (5 points) Assuming that the government balances its budget period-by-period, derive the amount of per-capita transfer ξ_t given τ_t.

   (b) (10 points) State the firm's profit maximization problem and derive it's first-order conditions.

   (c) (10 points) Derive the equilibrium conditions of this economy.

   (d) (10 points) Now we will introduce a preference shock as follows:

$$E \sum_{t=0}^{\infty} \beta^t \left[\ln c_t - \varphi_t \frac{h_t^{1+\eta}}{1+\eta} \right]$$

where

$$\varphi_{t+1} = (I - \rho_\varphi)\bar{\varphi} + \rho_\varphi \varphi_t + \epsilon_{\varphi,t+1}, \qquad \epsilon_{\varphi,t+1} \sim N(0, \sigma_\varphi^2).$$

with $\bar{\varphi} > 0$.

Assume that the hiring subsidy is no longer exogenous. Instead, the government wishes to set the subsidy in a way such that it completely offsets changes in φ_t. Moreover, in each period t, the government can set its τ_t after observing the realization of φ_t. Derive the expression that describes the optimal subsidy τ_t.