Homework 3
Econ 205B, Winter 2017

You are encouraged to discuss the problems in groups, but need to write down your own solutions.

1. The basic MIU model discussed in lecture implied that the marginal rate of substitution between money and consumption was set equal to $i_t/(1+i_t)$. That model assumed that agents entered period t with resources ω_t and used those to purchase capital, consumption, nominal bonds, and money. The real value of these money holdings yielded utility in period t. Assume instead that money holdings chosen in period t do not yield utility until period $t+1$. Utility is $\sum \beta^i U(c_{t+i}, M_{t+i}/P_{t+i})$ as before, but the budget constraint takes the form

$$\omega_t = c_t + \frac{M_{t+1}}{P_t} + b_t + k_t,$$

and the household chooses c_t, k_t, b_t, and M_{t+1} in period t. The household's real wealth ω_t is given by

$$\omega_t = f(k_{t-1}) + (1-\delta)k_{t-1} + (1+r_{t-1})b_{t-1} + m_t.$$

where $m_t = M_t/P_t$ and the household chooses c_t, k_t, b_t, and M_{t+1} in period t.

   (a) Derive the first order condition for the household's choice of M_{t+1} and show that

$$\frac{U_m(c_{t+1}, m_{t+1})}{U_c(c_{t+1}, m_{t+1})} = i_t$$

   (b) Explain why i and not $i/(1+i)$ appears on the right side of this condition.

2. Suppose utility depends on the consumption of two goods, denoted C_t^m and C_t^c. Purchases of C_t^m are subject to a cash in advance constraint; purchases of C_t^c are not. The two goods are produced by the same technology: $C_t^m + C_t^c = Y_t$, where Y_t is exogenously given.

   (a) Write down the household's decision problem.

   (b) Write down the first order conditions for the household's optimal choices for C_t^m and C_t^c. How are these affected by the cash in advance constraint?

   (c) Show that the nominal rate of interest acts as a tax on the consumption of C_t^m.

3. We consider two extensions to the baseline New Keynesian model we considered in class: capital accumulation and price indexation.

In each period t, the final goods, Y_t, are produced by a perfectly competitive representative firm that combines a continuum of intermediate goods, indexed by $j \in [0,1]$, with technology

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}.$$

$Y_{j,t}$ denotes the time t input of intermediate good j and θ controls the price elasticity of demand for each intermediate good. The demand function for good j is

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} Y_t,$$

where P_t and $P_{j,t}$ denote the price of the final good and intermediate good j, respectively. Finally, P_t is related to $P_{j,t}$ via the relationship

$$P_t = \left[\int_0^1 P_{j,t}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}.$$

The intermediate-goods sector is monopolistically competitive. In period t, each firm j rents $K_{j,t}$ units of capital stock from the household sector and buys $H_{j,t}$ units of aggregate labor input from the employment sector to produce intermediate good j using technology

$$Y_{j,t} = z_t K_{j,t}^{\alpha} H_{j,t}^{1-\alpha}.$$

z_t is the level of total factor productivity that follows

$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t,$$

where ϵ_t is i.i.d. distributed from a normal distribution with mean zero and variance σ^2.

Firms face a Calvo-type price-setting friction: In each period t, a firm can reoptimize its intermediate-goods price with probability $(1-\xi)$. If they cannot, they index their price according to the function of past inflation rate and steady-state inflation rate, $\pi_{t-1}^{\gamma} \pi^{1-\gamma}$ where π is the steady-state inflation rate and $0 \leq \gamma \leq 1$.

(a) Solve the cost-minimization problem faced by intermediate-goods firms and derive the expression for the marginal cost.

(b) Write down the profit-maximization problem by intermediate-goods firms.

(c) Derive the New Keynesian Phillips Curve for this model. Show that when $\gamma = 0$, the NKPC reduces to the one covered in the class.

(d) How does the inflation dynamics change when γ increases?

4. Walsh, Chapter 8, question 3, page 387.