Homework 2

Econ 205B, Winter 2017

You are encouraged to discuss the problems in groups, but need to write down your own solutions.

1. We consider the effects of government spending shocks for different specifications regarding how government spending enter into production. Assume that the level of government spending G_t follows an AR(1) process:

$$\ln G_t = (1-\rho_g)\ln \bar{G} + \rho_g \ln G_{t-1} + \epsilon_t,$$

where $\bar{G}$ is the steady-state level of government spending and $\bar{G}/\bar{Y} = g$. We set $\rho_g = 0.95, g = 0.2$. For simplicity, assume that the government balances budget every period and there is no distortionary taxes; the government spending is financed by lump-sum taxes to households. The rest of the model is identical to the standard RBC model we covered in class.

   (a) Assume that government spending is wasteful (it does not enter into the utility nor production).

      i. Derive the equilibrium conditions.

      ii. Use Dynare plot the IRF to a government spending shock. Also plot the government spending multiplier:

$$\frac{dY_t}{dG_t} = \frac{1}{g}\frac{\hat{Y}_t}{\hat{G}_t}$$

   (b) Assume that government spending substitutes for private investment:

$$K_t = (1-\delta)K_{t-1} + I_t + G_t.$$

      i. Derive the equilibrium conditions.

      ii. Use Dynare to plot the IRF to a government spending shock and the government spending multiplier. Why are the results different from (a)?

2. We will consider a slightly different variation of the benchmark home production RBC model of Benhabib, Rogerson and Wright (1991). First, households value their leisure time because of what they can do with it (it is not the residual time unoccupied by production). Second, home consumption enters the utility function separably from market consumption.

The household maximizes expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$$

where we specify

$$U(c_t, h_t) = \frac{c_t^{1-\gamma}}{1-\gamma} + \theta \frac{h_t^{1-\lambda}}{1-\lambda}$$

Output is produced in both the home and market sectors according to the following technologies:

$$Y_t = z_{m,t} K_{m,t-1}^{1-\alpha_m} (X_{m,t} N_t)^{\alpha_m}$$
$$H_t = z_{h,t} K_{h,t-1}^{1-\alpha_h} (X_{h,t}(1-N_t))^{\alpha_h}$$

where Y_t is market output, $K_{m,t-1}$ is market capital, $K_{h,t-1}$ is household capital, and N_t is the portion of labor's endowed time allocated to market activities. $z_{m,t}$ and $z_{h,t}$ are the technology shocks which follow

$$\ln z_{m,t} = \rho_m \ln z_{m,t-1} + \epsilon_{m,t}, \quad \epsilon_{m,t} \sim N(0, \sigma_m^2)$$
$$\ln z_{h,t} = \rho_h \ln z_{h,t-1} + \epsilon_{h,t}, \quad \epsilon_{h,t} \sim N(0, \sigma_h^2)$$

and $X_{m,t}$ and $X_{h,t}$ are deterministic technology progress which follow $X_{m,t} = \gamma_m X_{m,t-1}$ and $X_{h,t} = \gamma_h X_{h,t-1}$.

Note that home production equals home production period by period; thus investment requires the production of market goods. More specifically, if the evolution of each capital stock is denoted by

$$K_{m,t} = (1-\delta) K_{m,t-1} + I_{m,t}$$

and

$$K_{h,t} = (1-\delta) K_{h,t-1} + I_{h,t}$$

where $I_{m,t}$ is business investment, and $I_{h,t}$ is household investment, the resource constraint for market output is given by

$$Y_t = C_t + I_{m,t} + I_{h,t}.$$

(a) Define the sequential market equilibrium, including the household's problem and the firm's problem.

(b) Define the recursive competitive equilibrium, including the household's problem and the firm's problem. (For the time being, you can assume there is no growth.)

(c) Derive the equilibrium conditions of this economy.

(d) What are the restrictions on parameters that ensures the existence of balanced growth path where $Y_t, K_{m,t}, K_{h,t}, I_{m,t}, I_{h,t}, C_t, H_t$ grow at the common rate and market hours stay constant?

(e) Assume for now $\alpha_h = 1$. Show that this economy is observationally equivalent to a one-sector RBC model with a preference shock $\tilde{\theta}_t$ where the household preference is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \tilde{\theta}_t \frac{h_t^{1-\tilde{\lambda}}}{1-\tilde{\lambda}} \right].$$