Homework 1
Econ 205B, Winter 2017

You are encouraged to discuss the problems in groups, but need to write down your own solutions.

1. The original Kydland and Prescott paper (1982) contained a time-to-build specification of the investment technology. Specifically, assume the following. When households undertake 1 unit of investment at period t, only $1/2$ unit turns into capital goods in the same period and the rest turns into capital at the end of period $t + 1$. Other parts of the model is identical to the RBC model we covered in class.

   (a) Define the sequential market equilibrium, including the household's problem and the firm's problem.

   (b) Define the recursive competitive equilibrium, including the household's problem and the firm's problem.

   (c) Derive the equilibrium conditions of this economy.

   (d) Derive the log-linearized capital accumulation equation and compare it with the standard model.

2. In addition to the technology shock, we will introduce a preference shock to the RBC model we considered in class. Suppose households maximize the expected discounted utility where instantaneous utility is given by

$$\frac{(a_t C_t)^{1-\sigma}}{1-\sigma} - \varphi \frac{H_t^{1+\eta}}{1+\eta}.$$

   where σ, η, and φ are parameters. Assume a_t follows

$$\ln a_t = \rho_a \ln a_{t-1} + \epsilon_{a,t}, \qquad \epsilon_{a,t} \sim N(0, \sigma_a^2).$$

   (a) Define the sequential market equilibrium, including the household's problem and the firm's problem.

   (b) Define the recursive competitive equilibrium, including the household's problem and the firm's problem.

(c) Derive the log-linearized Euler condition from the representative agent's decision problem. Does it depend on $\hat{a}_t$? For a given expected real interest rate and level of future consumption, how does a positive realization of $\hat{a}_t$ affect C_t? How does your answer depend on ρ_a? Explain.

(d) Derive the log-linearized labor supply equation for this model. (Assume the representative agent sells labor services at a real wage w_t in a perfectly competitive labor market.) Does it depend on $\hat{a}_t$? For a given expected real interest rate and level of future consumption, how does a positive realization of $\hat{a}_t$ affect H_t? Explain.

(e) Explain intuitively the effect $\hat{a}_t$ has on the correlation between output and consumption, between consumption and employment, between employment and real wages. (That is, does a positive realization of $\hat{a}_t$ cause output and consumption to move together or in different directions, etc.).

3. Instead of the one-sector RBC model we covered in class, suppose consumption goods and capital goods are produced in two different sectors. The production functions in both sectors are given by

$$C_t = z_{1,t} K_{1,t}^{\alpha} H_{1,t}^{1-\alpha}$$
$$K_t = z_{2,t} K_{2,t}^{\gamma} H_{2,t}^{1-\gamma}$$

Capital and labor inputs in both sections have to satisfy

$$K_{t-1} = K_{1,t} + K_{2,t}$$
$$H_t = H_{1,t} + H_{2,t}$$

Also $z_{1,t}$ and $z_{2,t}$ follows a standard AR(1) process. The planner can allocate both capital and labor freely across the two sectors in the current period, that is, no time is required to relocate either capital or labor across sectors. Define the recursive social planner's problem for this economy.

4. Everything is as in the previous question, but now capital in both sectors is predetermined; that is, if the planner decides to move capital between the two sectors today, this change takes effect at the beginning of next period. Labor can still be reallocated instantaneously. Define the recursive social planner's problem for this economy.

5. Solve the basic RBC model we covered in class (stochastic technology and variable labor) using the value function iteration method. Plot the value function and the policy function for C, H, and K'.