Final 2016

1.(a) $\lambda_t = U_c$

$$\lambda_t = \beta E_t\left[\lambda_{t+1}(r_{t+1} + 1 - \delta)\right]$$

$$\lambda_t = \beta(1+i_t) E_t\left[\frac{\lambda_{t+1}}{1+\pi_{t+1}}\right]$$

$$\lambda_t = U_m + \beta E_t\left[\frac{\lambda_{t+1}}{1+\pi_{t+1}}\right]$$

(b) $$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \Big[U(C_t, x_t)$$

$$+ \lambda_t \Big\{ (r_t + 1 - \delta) K_{t-1} + (1 + i_{t-1}) \frac{b_{t-1}}{1+\pi_t}$$

$$+ \frac{m_{t-1}}{1+\pi_t} + \tau_t - C_t - K_t - b_t - m_t \Big\}$$

$$+ \mu_t \Big\{ \frac{m_{t-1}}{1+\pi_t} + \tau_t + (1 + i_{t-1}) \frac{b_{t-1}}{1+\pi_t} - b_t - x_t \Big\} \Big]$$

FONC w.r.t. C_t

$$\lambda_t = U_c.$$

FONC w.r.t. K_t

$$\lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} (r_{t+1} + 1 - \delta) \right].$$

FONC w.r.t. b_t

$$\lambda_t + \mu_t = \beta (1 + \hat{r}_t) \, \mathbb{E}_t \left[\frac{\lambda_{t+1} + \mu_{t+1}}{1 + \pi_{t+1}} \right]$$

FONC w.r.t. m_t

$$\lambda_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1} + \mu_{t+1}}{1 + \pi_{t+1}} \right]$$

FONC w.r.t. x_t

$$\mu_t = U_x.$$

Also note by combining these eqs.

$$1 + \frac{\mu_t}{\lambda_t} = 1 + \hat{r}_t.$$

- The Euler eq. for bond holding is now affected by marginal util. of real balances.

2. (a) Assume internal habit so households internalize the effect of C_t on future utility

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - bC_{t-1})^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} m_t^{1-b} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right.$$

$$\left. + \lambda_t \{ \text{ budget constraint } \} \right]$$

① FONC for C_t

$$\lambda_t = (C_t - bC_{t-1})^{-\sigma} - \beta b \mathbb{E}_t (C_{t+1} - bC_t)^{-\sigma}.$$

② FONC for B_t

$$\lambda_t = \beta(1+i_t)\,\beta E_t\left(\frac{\lambda_{t+1}}{1+\pi_{t+1}}\right).$$

Log-linearize.

① : Define $Q_t \equiv C_t - bC_{t-1}$

$$\lambda_t = Q_t^{-\sigma} - \beta b \mathbb{E}_t Q_{t+1}^{-\sigma}.$$

$$\lambda(1+\hat{\lambda}_t) = Q^{-\sigma}(1-\sigma\hat{Q}_t) - \beta b Q^{-\sigma}(1-\sigma E_t\hat{Q}_{t+1})$$

In the s-s, $\lambda = (1-\beta b)Q^{-\sigma}$.

$$\boxed{\lambda\hat{\lambda}_t = -\sigma Q^{-\sigma}(\hat{Q}_t - \beta b E_t \hat{Q}_{t+1})}$$

$$Q(1+\hat{Q}_t) = C(1+\hat{C}_t) - bC(1+\hat{C}_{t-1}),$$

$$Q\hat{Q}_t = C\hat{C}_t - bC\hat{C}_{t-1}$$

Using $C_t = Y_t$,

$$\boxed{Q\hat{Q}_t = Y\hat{Y}_t - bY\hat{Y}_{t-1}}$$

Log-linearize (2)

$$\boxed{\hat{\lambda}_t = \mathbb{E}_t\hat{\lambda}_{t+1} - (\hat{i}_t - \mathbb{E}_t\pi_{t+1})}$$

(b) Define output gap: $X_t \equiv \hat{Y}_t - \hat{Y}_t^f$.

$$Q\hat{Q}_t = YX_t - bYX_{t-1} + u_t,$$

where $u_t \equiv Y\hat{Y}_t^f - bY\hat{Y}_{t-1}^f$

(c). A surprise increase in interest rate reduces output gap. Because of cons. habit, the response of output gap is smoother.