Final Winter 2015

1.(a) Capitalist's problem

$$V^c(k^c, K^c, z)$$

(This can be K instead of K^c)

$$= \max_{c^c, h^c, i^c} \left[u(c^c, h^c) + \beta E V^c(k^{c'}, K^{c'}, z') \right]$$

$$\text{s.t. } c^c + i^c \leq w h^c + r k^c$$

$$k^{c'} = (1-\delta) k^c + i^c$$

Worker's problem

$$V^w(K^c, z) = \max_{c^w, h^w} \left[u(c^w, h^w) + \beta E V^w(K^{c'}, z') \right]$$

$$\text{s.t. } c^w = w h^w$$

RCE is (i) HH's value func,

$V^c(—)$, $V^w(—)$, policy func.

$c^c(—)$, $h^c(—)$, $i^c(—)$

$c^w(—)$, $h^w(—)$

(ii) firm's policy $H^d(\sim)$, $K^d(\sim)$

(iii) prices r ——

(iv) LOM of capital $K^{c'} = g(K^c, z)$

such that

- HH optimization

- Firm optimization

- Market clearing

$$\begin{cases} h = H = H^d \\ k = K = K^d \\ C + I = zF(K, H) \end{cases}$$

where $K = sK^c$, $H = sH^c + (1-s)H^w$,
$C = sC^c + (1-s)C^w$, $I = sI^w$.

- Consistency

$$g(K^c, z) = (1-\delta)K^c + i(K^c, K^c, z).$$

1.(b) Sequential social planner's problem

$$\max_{\substack{\{C^c_t, H^c_t,\\ C^w_t, H^w_t, I_t\}}} \mathbb{E} \sum \beta^t \Big[s\, u(C^c_t, H^c_t) + (1-s)\, u(C^w_t, H^w_t) \Big]$$

$$\text{s.t.} \quad \begin{cases} C_t + I_t \le z_t F(K_{t-1}, H_t), \\ K_t = (1-\delta) K_{t-1} + I_t \\ s C^c_t + (1-s) C^w_t = C_t \\ s H^c_t + (1-s) H^w_t = H_t \end{cases}$$

Note: Competitive eqm. allocation is different from the solution to planner's problem!

Social planner (with equal weights) will allocate same level of C and H to capitalists and workers.

This wouldn't happen in competitive eq...

(c) - Capitalist's FONC are the standard conditions.
(Euler eq. and labor supply condition)

- Workers' FONC is only one.

$$\frac{1}{c_t^w} w_t = (h_t^w)^\eta \quad \text{———} \quad ⊛$$

- Also note capitalists and workers face the same wage, which is equal to the marginal product of labor.

(d) Substitute workers' b.c. into ⊛

$$\frac{1}{\cancel{w_t} h_t^w} \cdot \cancel{w_t} = (h_t^w)^\eta$$

$$\rightarrow 1 = (h_t^w)^{1+\eta}$$

$\rightarrow h_t^w$ const. over business cycle.

This is because the util. function implies that income and substitution effects cancel out.

2.(a) See lecture note.

(b) See lecture note.

Note
$$
\begin{cases}
U_c = [C_\pi + m_\pi]^{-\sigma} \\
U_H = H_\sigma^{\eta} \\
U_m = [C_\pi + m_\pi]^{-\sigma}
\end{cases}
$$

(c) No. money is non-neutral in this economy

Since U_c affected by m, real side affected by change in money level.

3.(a) Forward-looking inflation process with real marginal cost as a driving force.

(b) $\lambda_t = d_t C_t^{-\sigma}$

$\Rightarrow \hat{\lambda}_t = \hat{d}_t - \sigma \hat{C}_t$

Also since $\mathbb{E}_t \hat{d}_{t+1} = \rho \hat{d}_t$

$\hat{\lambda}_t = \hat{i}_t - \mathbb{E}_t \pi_{t+1} + \mathbb{E}_t \hat{\lambda}_{t+1}$

$\hat{d}_t - \sigma \hat{C}_t = \hat{i}_t - \mathbb{E}_t \pi_{t+1} + \rho \hat{d}_t - \sigma \overset{\mathbb{E}_t}{\hat{C}}_{t+1}$

Also use $\hat{C}_t = \hat{Y}_t$. Divide both sides by $-\sigma$

$-\frac{(1-\rho)}{\sigma}\hat{d}_t + \hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{i}_t - \mathbb{E}_t \pi_{t+1}$

$\Rightarrow -\frac{(1-\rho)}{\sigma}\hat{d}_t + \hat{x}_t = \mathbb{E}_t \hat{x}_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{i}_t - \mathbb{E}_t \pi_{t+1}$

$+ u_t$

$u_t \equiv \mathbb{E}_t \hat{y}^{f}_{t+1} - \hat{y}^{f}_t$

(c) RHS const. Assume $\sigma > 0$.

$\Rightarrow \hat{y}_{\sigma}$ increases, when $\hat{d}_{r}$ increases.

(d) Since price sticky, real interest rate $\uparrow$

$\underbrace{\text{Output (gap) decreases}}_{\text{through Euler eq.}}$ and $\underbrace{\text{inflation decreases}}_{\text{Through NKPC}}$.