20513 W14 Final

Brief answers

1 (a). A SME is

(i) HH's policy functions

$c_m(k_{m,t-1}, k_{h,t-1}, K_{m,t-1}, K_{h,t-1}, \varphi_t, z_{m,t}, z_{h,t})$

$c_h(\text{———})$, $h_m(\text{——})$, $h_h(\text{——})$,

$i_m(\text{———})$, $i_h(\text{——})$.

(ii) Firm's policy func.

$H_m^d(K_{m,t-1}, K_{h,t-1}, \varphi_t, z_{m,t}, z_{h,t})$

$K_m^d(\text{————————})$.

(iii) Prices.

$w(K_{m,t-1}, K_{h,t-1}, \varphi_t, z_{m,t}, z_{h,t})$.

$r(\text{————————})$.

(iv) LOM for capitals.

$$\begin{cases} K_m = g_m(K_{m,t-1}, K_{h,t-1}, \varphi_t, z_{m,t}, z_{h,t}), \\ K_h = g_h(\text{————————}) \end{cases}$$

(v). government transfer func

$\xi(K_{m,t-1}, K_{h,t-1}, \varphi_t, z_{m,t}, z_{h,t})$ such that

- Given prices and LOM for capital, HH's policy func. solve the HH's problem:

$$\max_{\{c_{m,t}, c_{a,t}, h_{m,t}, h_{a,t}, i_{m,t}, i_{a,t}\}} \mathbb{E} \sum \beta^t u(\text{———}),$$

$$\text{s.t.} \begin{cases} c_{m,t} + i_{m,t} + i_{a,t} \leq \text{———}. \\ k_{m,t} = \text{———}. \\ k_{a,t} = \text{———}, \\ c_a = z_a (k_{a,t-1})^{\alpha_a} (h_{a,t})^{1-\alpha_a} \end{cases}$$

- Given prices, firm's policy solve firm's problem,

$$\max_{K^d_{m,t}, H^d_{m,t}} \left[z_{m,t} (K^d_{m,t})^{\alpha_m} (H^d_{m,t})^{1-\alpha_m} - r_t K^d_{m,t} - w_t H^d_{m,t} \right.$$

- Gov's B.C. is satisfied,
- Markets clear

$$\begin{cases} - \ h_{m,t} = H_{m,t} = H^d_{m,t} \\ - \ k_{m,t-1} = K_{m,t-1} = K^d_{m,t} \\ - \ c_{m,t} + i_{m,t} + i_{a,t} = z_{m,t} K_{m,t-1}^{\alpha_m} H_{m,t}^{1-\alpha_m}. \end{cases}$$

- Consistency.

$$g_m(\text{——}) = (1-\delta)K_{m,t-1} + i_m(K_{m,t-1}, K_{h,t-1}, K_{m,t-1}, K_{h,t-1}, \varphi_t, z_{m,t}, z_{h,t})$$

$$g_h(\text{——}) = (1-\delta)K_{h,t-1} + i_h(\text{——————})$$

(b) A RCE is

(i) HH's value func,

$V(k_m, k_h, K_m, K_h, \varphi, z_m, z_h)$.

policy func,

$c_m(\text{——})$, $c_h(\text{——})$, $h_m(\text{——})$, $h_h(\text{——})$

$i_m(\text{——})$, $i_h(\text{——})$,

(ii) Firm's policy func.

$H_m^d(K_m, K_h, \varphi, z_m, z_h)$,

$K_m^d(\text{——————})$,

(iii) Prices,

$w(K_m, K_h, \varphi, z_m, z_h)$,

$r(\text{——————})$,

(iv), LOM for capitals,

$$\begin{cases} K_m' = g_m(K_m, K_a, \varphi, z_m, z_a), \\ K_a' = g_a(\text{———}) \end{cases}$$

(v) goverment transfer func,

$\xi(K_m, K_a, \varphi, z_m, z_a)$ such that,

- Given prices and LOM for capital, HH's value and policy func solve the Bellman eq.

$$V(k_m, k_a, K_m, K_a, \varphi, z_m, z_a)$$

$$= \max_{\{c_m, c_a, i_m, i_a, h_m, h_a\}} \left[u(\text{———}) + \beta \mathbb{E} V(k_m', k_a', K_m', K_a', \varphi', z_m', z_a') \right]$$

s.t. $c_m + i_m + i_a \leq$ ———,

$k_m' =$ ———,

$k_a' =$ ———,

$c_a = z_a (k_{a,t-1})^{\alpha_a} (h_{a,t})^{1-\alpha_a}$

- Gov's B.C. is satisfied.

- Markets clean.

$$\begin{cases} \cdot\ h_m = H_m = H_m^d \\ \cdot\ k_m = K_m = K_m^d \\ \cdot\ C_m + i_m + i_a = Z_m K_m^{\alpha_m} H_m^{1-\alpha_m} \end{cases}$$

- Consistency.

$$\begin{cases} g_m(\text{---}) = (1-\delta)K_m \\ \qquad + i_m(K_m, K_a, k_m, k_a, \varphi, z_m, z_a) \\ g_a(\text{---}) = (1-\delta)K_a + i_a(\text{---}) \end{cases}$$

(c). <u>HH's problem</u>

$$\mathcal{L} = \mathbb{E}\sum \beta^t \Bigg[\frac{\left(b c_{m,t} + (1-b) \overbrace{z_{a,t} k_{a,t-1}^{\alpha_a} h_{a,t}^{1-\alpha_a}}^{c_{a,t}}\right)^{1-\sigma}}{1-\sigma} - \frac{(h_{m,t} + h_{a,t})^{1+\eta}}{1+\eta}$$

$$+ \lambda_t \Big\{ r_t k_{m,t-1} + (1-\varphi_t) w_t h_{m,t} + \xi_t - C_{m,t} - k_{m,t} + (1-\delta) k_{m,t-1} - k_{a,t} + (1-\delta) k_{a,t-1} \Big\} \Bigg]$$

Define $\begin{cases} c_t \equiv b\, c_{m,t} + (1-b)\, c_{n,t} \\ h_t \equiv h_{m,t} + h_{n,t} \end{cases}$

<u>FONC w.r.t. $c_{m,t}$</u>

$$\lambda_t = b\, c_t^{-\sigma}.$$

<u>FONC w.r.t. $h_{m,t}$</u>

$$\lambda_t (1-\varphi_t)\, w_t = h_t^{\eta}.$$

<u>FONC w.r.t. $k_{m,t}$</u>

$$\lambda_t = \beta\, \mathbb{E}_t \left[\lambda_{t+1} (r_{t+1} + 1 - \delta) \right].$$

<u>FONC w.r.t. $h_{n,t}$</u>

$$c_t^{-\sigma} (1-b)(1-\alpha_n) \frac{c_{n,t}}{h_{n,t}} = h_t^{\eta}.$$

<u>FONC w.r.t. $k_{n,t}$</u>

$$\lambda_t = \beta\, \mathbb{E}_t \Big[c_{t+1}^{-\sigma} (1-b)\, \alpha_n \frac{c_{n,t+1}}{k_{n,t}} + \lambda_{t+1} (1-\delta) \Big].$$

and then also impose firm's FONC, market clearing conditions, etc.

(d) The hours response will be larger with home production because now households substitute between market production & home production because the home production is not taxed.

2(a)

$$V(\omega_t, m_{t-1}, z_t) = \max_{c_t, k_t, b_t, m_t} \left\{ u(c_t) + \beta E_t \left[V(\omega_{t+1}, m_t, z_{t+1}) \right] \right\}$$

$$\text{s.t.} \quad c_t + m_t + b_t + k_t \leq \omega_t \qquad \text{-- } \lambda_t$$

$$z_t c_t \leq \frac{m_{t-1}}{\Pi_t} + \tau_t \qquad \text{-- } \mu_t$$

$$\omega_{t+1} = f(k_t) + (1-\delta) k_t + \tau_{t+1} + R_t a_t - \left(\frac{i_t}{\Pi_{t+1}}\right) m_t.$$

where $R_t = \dfrac{1+i_t}{\Pi_{t+1}}$, $a_t = m_t + b_t$.

(b).

<u>c_t</u>

$$u'(c_t) - \lambda_t - g_t \mu_t = 0.$$

<u>k_t</u>

$$\beta [f'(k_t) + 1 - \delta] \mathbb{E}_t [V_w (w_{t+1}, m_t, g_{t+1})] - \lambda_t = 0.$$

<u>b_t</u>

$$\beta R_t \mathbb{E}_t [V_w (w_{t+1}, m_t, g_{t+1})] - \lambda_t = 0.$$

<u>m_t</u>

$$\beta \left[R_t - \frac{i_t}{\pi_{t+1}} \right] \mathbb{E}_t [V_w (w_{t+1}, m_t, g_{t+1})]$$

$$+ \beta \mathbb{E}_t [V_m (w_{t+1}, m_t, g_{t+1})] - \lambda_t = 0.$$

From Envelope cond.

$$\begin{cases} V_w (w_t, m_{t-1}, g_t) = \lambda_t \\ V_m (w_t, m_{t-1}, g_t) = \left(\frac{1}{\pi_t} \right) \mu_t. \end{cases}$$

Tighter CIA constraint,

(c) $u'(c_\tau) = \lambda_\tau + \xi_\tau \mu_\tau$.

So when ξ_τ increases, consumption declines. (since u' increases).

Households will increase their money holding since they anticipate higher ξ in the future (Also future cons. likely to decline).

(d) Cons. in period τ declines because of the tighter CIA constraint.

Households will not adjust their money holdings because expected future ξ unchanged. (Also future cons. is ambiguous, depends on the realization of future ξ).

3 (a)

- Let Σ_u be the estimate of covariance matrix. $\left(\begin{bmatrix} u_{y,t} \\ u_{x,t} \end{bmatrix} \sim N(0, \Sigma_u)\right)$
- Then apply cholesky decomposition.

$$\Sigma_u = \tilde{B}\tilde{\Omega}_e\tilde{B}'$$

where $\tilde{B}$ is a lower triangular matrix.

- $\tilde{\Omega}_e$ is the covar matrix of i.i.d. shocks.
- So you can simulate output effect by using

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \tilde{A}\begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \tilde{B}\begin{bmatrix} e_{y,t} \\ e_{x,t} \end{bmatrix}$$

$$\begin{bmatrix} e_{y,t} \\ e_{x,t} \end{bmatrix} \sim N(0, \tilde{\Omega}_e).$$

A policy shock is a shock to $e_{x,t}$.

(b) Modify the model so that agents make decision <u>before</u> the realization of e_t:

$$
\begin{cases}
x_t = \underline{\mathbb{E}_{t-1}}\, x_{t+1} - \left(\frac{1}{\sigma}\right)\left(\underline{\mathbb{E}_{t-1}}\, i_t - \underline{\mathbb{E}_{t-1}}\, \pi_{t+1}\right), \\
\pi_t = \beta\, \underline{\mathbb{E}_{t-1}}\, \pi_{t+1} + \kappa x_t. \\
i_t = \phi_\pi \pi_t + \phi_x x_t + e_t.
\end{cases}
$$

(c).

$$
\min_{\{i_{t+i}, \pi_{t+i}, x_{t+i}\}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \Big\{ \left(\frac{1}{2}\right)\left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right)
$$

$$
+ \theta_{t+i}\left[x_{t+i+1} - x_{t+i+2} + \sigma^{-1}\left(i_{t+i+1} - \pi_{t+i+2}\right)\right]
$$

$$
+ \varphi_{t+i}\left[\pi_{t+i+1} - \beta \pi_{t+i+2} - \kappa x_{t+i+1}\right] \Big\}
$$

Set $\mathbb{E}_\tau(\Theta_{\tau+i}) = 0$. Then FONC

for $\pi_{\tau+i+1}$, $x_{\tau+i+1}$:

$$\begin{cases} \pi_{\tau+1} + \varphi_{\tau+1} = 0. \\ \mathbb{E}_\tau(\pi_{\tau+i+1} + \varphi_{\tau+i+1} - \varphi_{\tau+i}) = 0 & i \geq 0 \\ \mathbb{E}_\tau(\lambda x_{\tau+i+1} - \kappa\varphi_{\tau+i+1}) = 0 & i \geq 0. \end{cases}$$

(d)

$$\begin{cases} \mathbb{E}_\tau(\pi_{\tau+i+1} + \varphi_{\tau+i+1} - \varphi_{\tau+i}) = 0 \\ \mathbb{E}_\tau(\lambda x_{\tau+i+1} - \kappa\varphi_{\tau+i+1}) = 0. \end{cases}$$