Final
Econ 205B, Winter 2014

- *You have 90 minutes to complete the exam. The maximum points possible is 100.*
- *No question can be asked during the exam. If you are unsure about the question, state clearly your interpretation and answer appropriately.*
- *Be concise. Long answers with redundant statements, even if they contain correct answers, will likely be heavily penalized.*

1. We will introduce a labor income tax shock into the home production RBC model. Household maximizes utility

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_{m,t}, c_{h,t}, h_{m,t}, h_{h,t})\right]$$

subject to

$$c_{m,t} + i_{m,t} + i_{h,t} \leq r_t k_{m,t-1} + (1-\varphi_t)w_t h_{m,t} + \xi_t$$
$$k_{m,t} = (1-\delta)k_{m,t-1} + i_{m,t}$$
$$k_{h,t} = (1-\delta)k_{h,t-1} + i_{h,t}$$

where φ_t is the labor income tax rate that follows

$$\varphi_{t+1} = (I-\rho_\varphi)\bar{\varphi} + \rho_\varphi \varphi_t + \epsilon_{\varphi,t+1}, \qquad \epsilon_{\varphi,t+1} \sim N(0, \sigma_\varphi^2).$$

and ξ_t is the lump-sum transfer from the government.

We specify the utility function as

$$u = \frac{(bc_{m,t} + (1-b)c_{h,t})^{1-\sigma}}{1-\sigma} - \frac{(h_{m,t} + h_{h,t})^{1+\eta}}{1+\eta}$$

and the production functions as

$$Y_t = z_{m,t} K_{m,t-1}^{\alpha_m} H_{m,t}^{1-\alpha_m}$$
$$C_{h,t} = z_{h,t} K_{h,t-1}^{\alpha_h} H_{h,t}^{1-\alpha_h}$$

where $z_{m,t}$ and $z_{h,t}$ are AR(1) productivity shocks.

Finally, the resource constraint of this economy is

$$C_{m,t} + I_{m,t} + I_{h,t} = Y_t.$$

(a) (10 points) Define the sequential market equilibrium, including the household's problem and the firm's problem.

(b) (10 points) Define the recursive competitive equilibrium, including the household's problem and the firm's problem.

(c) (10 points) Derive the equilibrium conditions of this economy.

(d) (5 points) Explain the difference between the market hours response to a labor income tax shock in this model and the response in the RBC model without home production.

2. We will consider a CIA model where a fraction q_t of consumption must be purchased using cash. We assume that q_t follows an AR(1) process.[1]

Household maximizes utility

$$E \sum_{t=0}^{\infty} u(c_t)$$

subject to the CIA constraint and the budget constraint. Assume that the goods market opens first so that the household's CIA constraint is given by

$$q_t c_t \leq \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} = \frac{m_{t-1}}{\Pi_t} + \tau_t$$

where M is a nominal money holding, T is a lump-sum transfer, and $m_{t-1} = M_{t-1}/P_{t-1}$, $\tau_t = T_t/P_{t-1}$, $\Pi_t = P_t/P_{t-1}$. The budge constraint in nominal terms is

$$\begin{aligned} P_t \omega_t &\equiv P_t f(k_{t-1}) + (1-\delta) P_t k_{t-1} + M_{t-1} + T_t + (1 + i_{t-1}) B_{t-1} \\ &\geq P_t c_t + P_t k_t + M_t + B_t. \end{aligned}$$

where f is an income generated during period t. In real terms, $\omega_t \geq c_t + m_t + b_t + k_t$ where

$$\omega_t \equiv f(k_{t-1}) + (1-\delta) k_{t-1} + \tau_t + \frac{m_{t-1} + (1 + i_{t-1}) b_{t-1}}{\Pi_t}.$$

(a) (10 points) Write down the household's decision problem.

(b) (10 points) Write down the household's first-order conditions.

(c) (5 points) How do the household respond to an increase in q_t?

(d) (5 points) Suppose q_t is i.i.d. instead of AR(1). How do the household respond to an increase in q_t?

[1] Technically we also want to have $0 \leq q_t \leq 1$, but this can be achieved by an appropriate transformation of q_t so we will not worry about it now.

3. (a) (5 points) Consider a bivariate VAR(1) system:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = A \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix}$$

where y_t is a log-deviation of real output from trend and x_t is a short-term nominal interest rate. Explain briefly how to identify the output effect to a monetary policy shock under the assumption that the policy shock does not affect other endogenous variables contemporaneously.

(b) (10 points) Consider the baseline New Keynesian model:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - E_t \pi_{t+1})$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

$$i_t = \phi_\pi \pi_t + \phi_x x_t + e_t$$

where $e_t \sim N(0, \sigma^2)$ is a monetary policy shock. How should we modify the model to make it consistent with the timing assumption we used in the VAR?

(c) (10 points) Suppose the central bank's objective is to minimize the loss function:

$$L_t = \left(\frac{1}{2}\right) E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2)$$

Under the model derived in the previous question, derive the first-order conditions for the *fully* optimal commitment policy.

(d) (10 points) Everything is the same as the previous question, but now derive the first-order conditions for the optimal commitment policy from a timeless perspective.