Economics 205A
Fall 2016
K. Kletzer

**Problem Set 5**
Due: Friday, December 2, 2016

1. Consider the optimal capital income tax problem for the case in which household utility is given by

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\sigma}}{1-\sigma} + \log(1-n_t) \right)$$

The production function is the standard constant returns to scale one, $y = f(k, n)$.

a) Set up the optimization problem for the household with proportional taxes on capital and labor income. Write down the first-order conditions for the household optimum.

b) Set up the optimal tax problem for the government.

c) Derive the first-order conditions for the government's optimal policy.

d) Show (or explain) that the optimal capital income tax will equal zero for all $t > 1$. That is, the only positive tax on capital income for these preferences can be $\tau_1^k$ if we rule out a capital levy at time $0$ (that is, $\tau_0^k = 0$ is fixed).

2. Consider the optimal tax problem outlined in the reading, but assume the production function, $y_t = wn_t$, where $w$ is constant labor productivity. There is no capital in this economy, and the only financial asset is government debt. Let household utility be given by

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left( \log c_t + \log(1-n_t) \right)$$

a) Assume the government only taxes labor. Write down the household problem and the first-order conditions for its optimum.

b) Write down the government's optimal tax problem (note the simplicity of the resource constraint with no capital). Allow for time-varying expenditures, $g_t$. Simplify the implementability constraint using this particular utility function and production function.

c) Using your first-order conditions for the optimal policy, the resource constraint and household first-order conditions, you can derive an expression for the optimal labor income tax rate in terms of the multiplier on the implementability condition and leisure consumption. (Hint: The expression you derive will be $\tau_t^n = \frac{\mu}{\mu + (1-n_t)}$ where $\mu$ is the multiplier on the implementability constraint.)

d) Suppose $g_t$ is constant for all $t$. Is $\tau_t^n$ constant under the optimal tax policy? $\mu$ is the shadow price of distortionary taxation. Does this rise with $g$? Does this make sense to you?

3. Consider an overlapping generations economy with two-period lived households. The utility function for the typical household born in period $t$ is given by

$$U_t = \log c_t^t + \beta \log c_{t+1}^t,$$

where $c_t^t$ denotes consumption of the generation born at $t$ in period $t$, and $c_{t+1}^t$ denotes its consumption in period $t+1$. Each generation only works when young supplying a unit of labor inelastically. Output per worker is given by the concave production function,

$$y_t = f(k_t),$$

and the rate of depreciation is zero. Let the population grow at the constant proportionate rate $\eta > 0$.

a) Find the conditions that determine the decentralized equilibrium given an initial capital stock, $k_0 > 0$.

b) Find the steady state for this economy.

c) Introduce an unfunded social security system. Let the lump-sum tax on the young at date $t$ be given by $T_t^t$ and the lump-sum tax imposed on the same generation at time $t+1$ when old be $T_{t+1}^t$. If the old receive a transfer, then $T_{t+1}^t$ is negative. The government budget is balanced for $g_t = 0$ if

$$T_t^t + \frac{T_t^{t-1}}{1+\eta} = 0.$$

Rewrite your equilibrium conditions with this policy included.

d) Let the policy set constant per capita taxes on the young, $T_t^t = T_1$, for all $t \geq 0$. Suppose this policy is introduced at time $0$ (so that $T_t^t = 0$ for $t = -1$). Show that the introduction of this policy reduces saving by every generation born at time $t \geq 0$.

4. Continue with the economy of problem 3.

a) Introduce public debt. Use the notation $b_t$ to denote public debt per capita of the young. This is $B_t/N_t$ where $N_t$ is the number of young born at $t$. Verify that the government budget identity is given by

$$(1+\eta)\, b_{t+1} = (1+r_t)\, b_t + g_t - \left(T_t^t + \frac{T_t^{t-1}}{1+\eta}\right).$$

Write out the conditions that determine the decentralized equilibrium letting $g_t = 0$ for all $t$.

b) Let the economy be in the steady state with $b_0 = 0$. The government makes a transfer to the currently old (the generation born at $t = -1$ receives the transfer $-T_0^{-1} > 0$ at $t = 0$). This is financed by imposing a constant lump-sum tax on the young, $T_t^t$, for all $t \geq 0$. Let $T_{t+1}^t = 0$ for all $t \geq 0$. The per capita tax, $T_t^t$, is set so that public debt per capita remains constant: $b_{t+1} = b_t$. Show that this policy reduces $k_{t+1}$ for all $t \geq 0$.

c) In this economy, the steady state marginal productivity of capital, $f'(k^*)$, can be less than $\eta$. Suppose this is the case. Show that the tax policy in part b increases the utility of the generation that is old at time $0$ and increases the utility of the young born at every date, $t \geq 0$. Explain how this is possible.