Economics 205A
Fall 2016
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Problem Set 4

Due: Tuesday, November 22, 2016

1. In this problem, we add government to the decentralized economy. Output is produced acording to the production function, $y_t = f(k_t)$, and capital depreciates at a constant rate, δ. Let $\{g_t\}_{t=0}^{\infty}$ be an exogenously given stream of government expenditures. The representative household utility is given by $U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(c_s)$. The government finances its expenditures by imposing lump-sum taxes on households.

a) Define a competitive equilibrium beginning with the household optimization problem and the firm optimization problem. Write out the market equilibrium conditions using the government budget identity as

$$b_{t+1} = (1 + r_t) b_t + g_t - T_t.$$

b) Use the conditions that determine competitive equilibrium to show that $\{c_t, k_{t+1}\}_{t=0}^{\infty}$ depends on $\{g_t\}_{t=0}^{\infty}$ and k_0 but not on the sequence of lump-sum taxes, $\{T_t\}_{t=0}^{\infty}$. You should find that only the path of b_{t+1} (and $a_{t+1} = b_{t+1} + k_{t+1}$) depends on $\{T_t\}_{t=0}^{\infty}$.

c) Use the government budget identity and solvency condition to solve for the government's intertemporal budget constraint. Observe that the government budget constraint is the only restriction on the path of taxes, so that the timing of taxes has no effect on household utility or the growth of the economy. Note that the full sequence of government expenditure, $\{g_t\}_{t=0}^{\infty}$, does matter for capital accumulation, household consumption and welfare.

2. A simple way to depict productive public spending is to allow public goods to enter production in an endogenous growth model. Let the production function for firms be

$$y_t = A k_t^{\alpha} g_t^{1-\alpha}$$

and let the rate of depreciation equal δ. Assume g_t is government consumption (that is, not investment). Let household utility equal

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \quad \text{for} \quad u(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

to allow a closed-form solution. The two factors of production are capital and government expenditures; there is no labor in this economy. The representative household owns all firms.

a) For a given fiscal policy with lump-sum taxation, find the relationships that determine a decentralized equilibrium. You will need to include profit income paid to households in the budget identity.

b) Solve out for equilibrium for the case that g_t equals some constant proportion of the capital stock ($\frac{g_t}{k_t} > 0$ is constant). What is the marginal product of capital and growth rate in the balanced growth path?

c) Find the command optimum by including g_t in the control set. How does g_t depend on k_t? What is the marginal product of capital and growth rate in the optimum? (Is the optimum a balanced growth path for all $t \geq 0$?)

d) How should we interpret equilibrium profits? How are these related to g_t in the optimum? Confirm that a balanced budget lump-sum tax is identical to a one hundred percent tax on profit.

3. In this problem, the government collects revenue by taxing labor earnings. The household budget identity becomes

$$a_{t+1} = (1 + r_t)\, a_t + (1 - \tau_t)\, w_t n_t - c_t,$$

where τ_t is the proportionate tax on labor income, $w_t n_t$, and n_t is household labor supply. The household utility is

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u\left(c_s, 1 - n_s\right).$$

a) Find the necessary conditions for household optimum.

b) Write out the government budget identity and solvency condition. Let government expenditures be given by the sequence $\{g_t\}_{t=0}^{\infty}$.

c) Add firms and the resource identity for the neoclassical growth model. Define a decentralized equilibrium for this economy. Write out the conditions that determine equilibrium.

4. In this problem, consider a specific example of labor income tax model of problem 3. The household utility function is

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{c_s^{1-\sigma}}{1-\sigma} - \frac{n_s^{1+\gamma}}{1+\gamma} \right)$$

for $0 \leq n_s \leq 1$.) We will just focus on how taxes affect household behavior.

a) Assume the household faces a constant interest rate, $r = \rho$. Write out the necessary conditions for a household optimum.

b) Consider an equilibrium with constant $g_t = g$ such that the tax rate, τ_t, is constant for all $t \geq 0$. You should be able to determine c_0 as a function of either g or the tax rate.

c) Suppose instead that government expenditure equals g for all $t \geq 0$ but the tax rate equal 0 for $t = 0$ to $t = T$. Beginning at $t = T + 1$, the tax rate rises to a constant τ' such that the government is solvent. How does labor supply change between $t = T$ and $t = T + 1$? Compare consumption at time 0 under this policy with the policy in part b.

d) Use your answers to parts b and c to explain how the timing of taxes matters with a labor income tax and endogenous labor supply.