Economics 205A
Fall 2016
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Problem Set 3

Due: Tuesday, November 2

1. Consider a household that maximizes a utility over consumption given by $U_0 = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}-1}{1-\sigma}$. The household receives a wage rate, w_t, for labor which it supplies perfectly inelastically as one unit each date. It also saves by accumulating financial assets, a_{t+1}, which earn a rate of return, r. The household budget identity is

$$a_{t+1} - a_t = r_t a_t + w_t - c_t,$$

and initial financial wealth is denoted a_0. The solvency constraint is also imposed.

a) Set up the household optimization problem when it takes $\{r_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$ as given.

b) Derive the necessary conditions for household optimum.

c) Show that the transversality and Euler conditions imply that the solvency constraint holds with equality.

d) Use these conditions to solve for consumption at date 0 as a function of wealth at date 0.

e) Simplify your result for the case in which $r_t = r$ is constant. Compare how household consumption and saving behave over time when $\rho > r$ and when $\rho < r$.

2. Consider household consumption and saving when utility is given by

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where the budget identity is

$$a_{t+1} - a_t = r a_t + w_t - c_t$$

and $r = \rho$.

a) Suppose that $w_0 = \bar{w} + \varepsilon$ and $w_t = w$ for all $t > 0$. Show how consumption and saving at any time $t = 0$ depends on the magnitude of ε. Compare this answer to the case in which $w_t = w + \varepsilon$ for all$t \geq 0$.

Interpret your results in terms of temporary and permanent unanticipated increase in labor income.

b) Suppose that labor income follows the process $w_{t+1} - w = \theta(w_t - w)$ and $w_0 = w + \varepsilon$. Let $1 > \theta \geq 0$. Show how consumption and saving at time $t = 0$ depends on ε. The parameter θ gives the persistence of the increase in labor income at time 0. Explain how an increase in θ affects consumption at time 0.

c) Suppose now that the household learns (at time 0) that its labor income will rise at a future time, $T > 0$, from w to $w + \varepsilon$. For simplicity, assume that earnings equal $w + \varepsilon$ for all $t \geq T$. How do consumption and saving respond to the new information at time 0?

3. Consider the household optimization problem when labor supply is endogenous using the utility function

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t),$$

for $u(c_t, \ell_t) = \log c_t + \frac{\ell_t^{1-\gamma}}{1-\gamma}$, where ℓ denotes leisure consumption. The endowment of leisure time equals 1, and leisure time can also be sold as labor time, n_t, at a wage rate w_t. Let $r_t = r$ be constant.

a) Set up the household optimization problem for this case allowing the wage rate to vary.

b) Derive the necessary conditions for household optimum.

c) Let $r = \rho$, and let $w_t = w$, where $w > 0$ is constant. Derive an equation that determines c_0 in terms of financial wealth and the present value labor income. Your equation will determine c_0 implicitly; you will not be able to write a closed form solution for c_0 unless $\gamma = 1$. Give a similar equation for n_0.

d) Suppose now that $w_t = w$ for all t such that $0 \leq t \leq T - 1$ but rises permanently to $w_t = w + \Delta w$ at time T. At time 0, the household knows that w_t will rise at time T. Show how this anticipated increase in wages affects household consumption, labor supply and saving at date 0. (As in part d, do not try to place just c_0 on the left-hand side, and continue to let $r = \rho$.)

4. This problem is about the optimization problem for a representative firm. The technology available to the economy is described by the production function, $y = f(k)$, where y denotes output per worker (a unit of labor) and capital per worker. For the economy, we can assume that labor supply is fixed, but we must allow the firm to choose the quantity of labor it employs in any period. The production function for a representative firm is

$$y_t = n_t f\left(\frac{k_t}{n_t}\right).$$

Here, y_t is the firm's output per capita of the population, n_t is the firm's employment as a proportion of the population and k_t is the capital used by the firm as a proportion of the population. Capital depreciates at the rate δ.

a) Suppose that the firm accumulates capital by issuing debt as in Wickens, section 4.7.1. Starting from the expression for the value of the firm, rewrite the value of the firm as the sum of two terms, the present value of $b_{t+1} - (1 + r) b_t$ for all $t \geq 0$ and the present value of output net of labor payments and investment, $y_t - w_t n_t - k_{t+1} + (1 - \delta) k_t$ for all $t \geq 0$. Next, evaluate the present value of $b_{t+1} - (1 + r) b_t$ (note the assumption you make when you calculate this sum).

b) From part a, you should get an expression for the quantity $V_0 + (1 + r) b_0$, the value of the firm, V_0, gross of the value of its initial debt, $(1 + r) b_0$. (In the notation used in the reading, $b > 0$ is a debt for the firm.) Now, use the first-order conditions for an optimum and the assumption that the firm uses a constant returns to scale technology to show that the value of the firm equals $(1 + r)(k_0 - b_0)$. You should pause to think about this result.

c) Instead of assuming the firm borrows to buy capital, we can assume that households own capital and rent it to firms that produce goods. The value of the capital owned by a representative household is

$$V_0^k = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t (\nu_t k_t - \delta k_t - \Delta k_{t+1})$$

where ν_t is the rent charged to firms per unit of capital, δk_t is the cost of depreciation and $k_{t+1} - k_t$ is the cost of increasing the capital stock. The profit for a firm is now

$$\pi_t = y_t - w_t n_t - \nu_t k_t.$$

In equilibrium, $\nu_t = r + \delta$, so assume this. Write down the value of the firm when it rents capital. Use this to show that when the firm maximizes its value, $V_0 = (1 + r) k_0$, where k_0 is interpreted as the initial investment made by the owners of the firm.