Economics 205A
Fall 2016
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Problem Set 2

Due: Tuesday, October 18, 2016

1. This problem examines how a temporary productivity shock affects the optimal growth solution. The production function is given by $y_t = A_t f(k_t)$ for $f(k)$ strictly concave. The planner's objective is household welfare given by $U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t)$ for strictly concave $u(c)$. For all times before and after time $t = 0$, productivity A_t equals a constant $A > 0$. For $t = 0$ only, $A_0 = A + \Delta A$ for $\Delta A > 0$.

a) Write out the linearized solution for the optimum in a neighborhood of the steady state for the economy when $A_t = A$ for all t. Denote the steady-state capital-labor ratio as k^* and steady-state consumption as c^*.

b) At time $t = 0$, let k_0 equal k^*. Write down the resource constraint $t = 0$ with $A_0 = A + \Delta A$. Linearly approximate this constraint in c_0 and k_1 at the steady state, k^* and c^* (that is, $\Delta c_0 = c_0 - c^*$ is the deviation in consumption at time 0 from c^* due to ΔA;similarly $\Delta k_1 = k_1 - k^*$). You will get a simple equation relating Δc_0, Δk_1 and ΔA.

c) Write down the Euler condition for $t = 0$ and linearly approximate it at the steady state to get a linear equation relating Δc_0, Δc_1 and Δk_1.

d) Your answer to part a gives you a relationship between Δc_1 and Δk_1. Solve the three linear equations (from parts a, b and c) to find a solution for Δc_0, Δc_1 and Δk_1 in terms of ΔA (in infinitesmals, the equations determine $\frac{dc_0}{dA}$, $\frac{dc_1}{dA}$ and $\frac{dk_1}{dA}$).

e) Wickens (page 32) claims that a temporary productivity shock causes c to rise for one period and then the economy just goes back to where it was (the steady state). Does our solution confirm or contradict this? Draw a phase diagram (this depicts the dynamics about the steady state so it uses the long-run production function $y_t = Af(k_t)$) to illustrate the solution we get. Explain why both c_0 and k_1 rise above the steady state and why k_t and c_t are larger than k^* and c^* for $t \geq 1$ as they converge back to the steady state over time.

2. This problem uses the linear economy. Let the production function be given by $y = Ak$ with a constant rate of depreciation δ. The planner's objective is household utility given by

$$U_0 = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma},$$

for $\sigma > 0$.

a) Set up and solve the problem for determining the optimum.

b) Consider the transversality condition and the general solutions for the dynamics for arbitrary parameters satisfying $A - \delta > 0$, $\rho > 0$ and $\sigma > 0$. Do you need any further restrictions on the parameters, A, δ, ρ and σ to ensure that the transversality condition is satisfied for some solution to the two-dimensional dynamics for c and k?

c) Use the solution for the optimum starting with some $k_0 > 0$. Substitute this into the utility function and check if the sum is finite. Do you need any restriction on the parameters A, δ, ρ and σ

to ensure that it is finite? Compare the restriction on a combination of these parameters (it's a simple formula) for parts c and b. What did you find?

d) You do not need to write out an answer to this part. Can you understand why the same restriction on parameters determines whether utility is bounded and whether the transversality condition holds in the solution to the first-order conditions for an optimum?

3. Allow labor supply to be endogenous by letting household utility be

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t),$$

where ℓ denotes leisure consumption. Impose the constraints that labor supply, n_t, and leisure consumption, ℓ_t, must each be nonnegative and that $0 \leq n_t + \ell_t \leq 1$. The production function is $y_t = Af(k_t, n_t)$ where k_t denotes capital per capita.

a) Write down the planner's problem for finding an optimum and write down the necessary conditions for the optimum.

b) Use the specific utility function,

$$u(c_t, \ell_t) = \log c_t + \frac{\ell_t^{1-\gamma}}{1-\gamma},$$

for $\gamma > 0$, to simplify the necessary conditions. Determine the steady state and show how you linearize the dynamics about the steady state.

c) How do the steady-state consumption, labor supply and capital stock vary with productivity A?

d) Suppose the economy starts with $k_0 < k^*$ near the steady state. How do you determine c_0 and n_0? How do c_t, k_t and n_t vary together along the optimum?

4. This problem completes the analysis of Tobin's q in Wickens, section 2.7.1 beginning with equations 2.40 and 2.41 on page 39. Please ignore equation 2.42 and the sentence just preceding it. This is one of errors in this book.

a) Write down the linearized model using equations 2.40 and 2.41. Rewrite these so that you have equations of motion for q and for k separately (so that Δk_{t+1} appears only in one equation and Δq_{t+1} only in the other). Determine the signs of the eigenvalues and of the slopes of the eigenvectors. Confirm that the dynamics in q and k are saddle-stable for strictly concave $f(k)$. For k_0 less than k^* near the steady state, confirm that $q > q^*$ and that net investment is positive at time $t = 0$.

b) Consider a permanent increase in productivity from A to $A + \Delta A$ when the economy starts in the steady state at time $t = 0$. How do q_0, i_0, and c_0 change?

c) Suppose productivity rises from A to $A+\Delta A$ for just one period at time $t = 0$ ($y_0 = (A + \Delta A) f(k^*)$ and $y_t = Af(k_t)$ for all $t \geq 1$). How do q_0, i_0, and c_0 change in this case? To answer this, you will need to refer to the original, nonlinearized, necessary conditions. The steady-state k^* does not change, but should $k_1 > k^*$?