Economics 205A
Fall 2016
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Problem Set 1

Due: Tuesday, October 4, 2016

1. Consider the simple command economy with capital in which the planner seeks to maximize the utility function

$$U_0 = \sum_{t=0}^{T} \beta^t u(c_t)$$

given the resource identity

$$k_{t+1} - k_t = f(k_t) - \delta k_t - c_t$$

given the initial capital-labor ratio, k_0 .

(a) Begin by setting up a Lagrangian and deriving the necessary conditions for an optimum.

(b) Now, let T go to infinity. What changes appear in the necessary conditions?

(c) Find the conditions that determine the steady state.

(d) Let $u(c) = \log c$. Write out the dynamics for k_t and c_t and linearize these around the steady state. Write out expressions for the eigenvalues.

(e) Find the eigenvectors associated with each eigenvalue (please use λ to denote an eigenvalue rather than repeat a messy expression). Is the steady state saddle-path stable? Explain.

2. Continue using this model, but change the production function to $y_t = A_t f(k_t)$ for $A_t > 0$.

(a) If A is constant, how does the steady state depend on the magnitude of A ?

(b) How does the solution for the linearized optimal growth path starting from some k_0 in a neighborhood of the steady state depend on A ?

(c) Use a phase diagram to illustrate your answer to part (b) by comparing the solution for some $A > 1$ to that for $A = 1$.

(d) Suppose the economy begins in steady state for $A = 1$ at time $t = 0$. Denote this allocation by $(\bar{c}_0, k_0)$. At $t = 0$, A permanently rises by some $\Delta A > 0$. Show and explain how consumption changes at time zero from $\bar{c}_0$ to c_0 on the phase diagram, then show how you would determine this analytically. Can you determine whether, or under what conditions, $c_0 > \bar{c}_0$ or $c_0 < \bar{c}_0$?

3. Consider the simple resource identity,

$$k_t - k_{t-1} = Ak_{t-1} - c_{t-1},$$

for $A > 0$ and constant.

(a) Write down the same relationship for k_{t-1} , and then substitute this expression into the one given above for k_t . By repeated substitution, you will arrive at a relationship between k_t and k_0 and a series in c_s for $s = 0, \ldots, t-1$.

(b) Use the same equation, but solve it forward in time. Start with

$$k_{t+1} - k_t = Ak_t - c_t,$$

where k_t is now given, and rearrange as

$$\frac{k_{t+1}}{1+A} = k_t - \frac{c_t}{1+A}.$$

Iterating forward in t , you solve for $(1+A)^{-T} k_{t+T}$ for any $T > 0$.

(c) Let T go to infinity. Show that you now have a constraint for the planner's consumption over time given that $k_t \geq 0$ for all $t \geq 0$.

(d) Observe that the set of consumption sequences, $(c_0, \ldots, c_t, ..)$, bounded by the constraints $k_t \geq 0$, $c_t \geq 0$ for all $t \geq 0$ is non-empty, compact and convex. Make sure you understand this. Now, maximize the utility function given by

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

with respect to $(c_0, \ldots, c_t, ..)$ subject to the constraint you derived in part (c). Derive the necessary conditions for an optimum.

4. In this problem, you will leave in the state variable k_t and use the sequence of resource identities,

$$k_{t+1} - k_t = Ak_t - c_t,$$

to find the optimum. The initial capital stock is given by k_0 .

(a) Use the Lagrangian to find all the necessary conditions for the optimum. Notice that you get the same conditions for the optimum using this method as you did in problem 3.

(b) Let $u(c_t) = \log c_t$ and use the necessary conditions to solve for consumption at $t = 0$, c_0 . You will be able to get a closed-form expression for this problem.

(c) Explain where you use the transversality condition in part (b).