Economics 205C
Spring 2016

Problem Set 4: Answers

1. Assume firms have a limited ability to monitor their workers. There are a large number of workers $\overline{L}$. Workers maximize expected discounted utility.

$$U_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(t)$$

where

$$u(t) = \begin{cases} w(t) - e(t) \text{ if employed} \\ b(t) \text{ if unemployed} \end{cases}$$

e is effort: e is either 0 or $\overline{e} > 0$. The term $b(t)$ is non-labor income (i.e., income independent of employment status). Workers are either working and exerting effort (E), working and shirking (S), or unemployed (U). Each period workers face an exogenous probability $\delta > 0$ of losing their job. An employed worker who shirks faces a probability q each period of being detected and fired. The probability an unemployed worker finds a new job is a per period. Unemployed workers receive an unemployment benefit b per period.

(a) Write down the value of each state the worker can be in. *The three states are working exerting effort (E), working and shirking (S), or being unemployed (U). Defining* $(1+\rho)^{-1} = \beta$, *the valuation equations are*

$$\rho V_E = (w - \overline{e}) + \delta (V_U - V_E)$$
$$\rho V_S = w + (\delta + q)(V_U - V_S)$$
$$\rho V_U = b + a (V_E - V_U)$$

This form parallels the class presentation of the Stiglitz-Shapiro model. Alternatively, think of V_E *as the value of state* E *at the end of* $t-1$. *Then*

$$V_E = \beta \left[w - e + \delta V_U + (1-\delta) V_E \right].$$

This can be rewritten as

$$\frac{1}{\beta} V_E = (1+\rho) V_E = w - e + \delta V_U + (1-\delta) V_E.$$

Subtracting V_E *from both sides, this becomes*

$$\rho V_E = w - e + \delta (V_U - V_E),$$

which is the form used above.

(b) Does difference in the value of being employment relative to unemployed depend on $b(t)$? Explain. *It does. From the valuation equations,*

$$\begin{aligned}\rho\left(V_E - V_U\right) &= \left[(w - \bar{e}) + \delta\left(V_U - V_E\right)\right] - \left[b + a\left(V_E - V_U\right)\right] \\ &= w - \bar{e} - b - (a + \delta)\left(V_E - V_U\right),\end{aligned}$$

or

$$V_E - V_U = \frac{w - \bar{e} - b}{\rho + a + \delta}.$$

The unemployment benefit reduces the value to the worker of being employed relative to being unemployed, for a given wage. But see part (c)..

(c) Derive the no shirking condition. What is the equilibrium wage? Does it depend on $b(t)$? *The no shirking condition (NSC) is $V_E = V_S$. From part (a),*

$$\begin{aligned}\rho\left(V_E - V_S\right) &= \left[(w - \bar{e}) + \delta\left(V_U - V_E\right)\right] - \left[w + (\delta + q)\left(V_U - V_S\right)\right] \\ &= -\bar{e} - \delta\left(V_E - V_S\right) - q\left(V_U - V_S\right).\end{aligned}$$

Since the NSC requires that $V_E = V_S$, this expression becomes

$$0 = -\bar{e} - q\left(V_U - V_E\right) \Rightarrow V_E - V_U = \frac{\bar{e}}{q}.$$

Written in this form, it is independent of b. The equilibrium wage is, using part (b),

$$V_E - V_U = \frac{w - \bar{e} - b}{\rho + a + \delta} = \frac{\bar{e}}{q}$$

or

$$w = \bar{e} + b + (\rho + a + \delta)\frac{\bar{e}}{q}.$$

The equilibrium wage does depend on b; the wage rises one-for-one with the unemployment benefit, for given a, and so the worker essentially received b whether working or unemployed..

(d) What is the per period flow into unemployment? What is the flow out? What is the equilibrium value of a? *The flow into unemployment in equilibrium (i.e., with no shirking occurring), is δN if N is total employment across all firms (note – in lecture total employment was denoted by NL where N was the number of firms). The flow out of unemployment is $a\left(\bar{N} - N\right) = aU$, where U denotes total unemployment and $\bar{N}$ is the total labor supply. In equilibrium, the flow out must equal the flow in, or*

$$a = \frac{\delta N}{U}.$$

(e) Suppose firms produce output according to $F\left(e(t)N(t)\right) = A\left[e(t)N(t)\right]^{\alpha}$, where $0 < \alpha \leq 1$ and $N(t)$ is per-firm employment. Firm profits are

$$A\left[e(t)N(t)\right]^{\alpha} - w(t)N(t),$$

where we have imposed the result that the wage ensures all employees exert effort. Using the equilibrium wage you found in part (c) and the assumption firms maximize profits, solve for the equilibrium employment and wages. How does a rise in $b(t)$ affect the equilibrium? Explain. *The profit maximizing firm's first-order condition is*

$$Ae^{\alpha}N^{\alpha-1} = w.$$

Hence,

$$N = \left[\frac{Ae^{\alpha}}{\bar{e} + b + (\rho + a + \delta)\frac{\bar{e}}{q}}\right]^{\frac{1}{1-\alpha}}.$$

This gives a negatively sloped relationship between a and N. The easier it is to find a job (i.e., the higher is α), the higher the wage must be to induce effort, and the lower will be the firm's demand for labor. From part (d),

$$a = \delta\frac{N}{\bar{N} - N},$$

which defines a positively sloped relationship between a and N. There intersection gives the equilibrium employment and job finding rate. A rise in b reduces employment (it increases the wage, leading to a reduced demand for employment. This is partially offset by the resulting fall in a which reduces the wage needed to satisfy the NSC.

2. Consider a standard new Keynesian model with sticky prices and wages; both adjust according to a simple Calvo model but with different degrees of stickiness.

   (a) What are the driving variables for price inflation and wage inflation? *From*

$$\begin{aligned}\pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa_p (\omega_t - mpl_t) \\ &= \kappa_p \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i (\omega_{t+i} - mpl_{t+i})\end{aligned}$$

   it is clear the driving variable for price inflation is $\omega_t - mpl_t$, i.e., the real wage relative to the marginal product of labor, which is real marginal cost. (Inflation at time t depends also on expected future inflation, but in solving the equation forward, we can see that real marginal cost is the true driving variable.) Firms wish to set their price relative to the general level of prices as a markup over real marginal cost. So if real marginal cost rises, firms that can adjust will raise their price. Similarly, for wage adjustment workers compare the real wage to the marginal rate of substitution between leisure and consumption, so $mrs_t - \omega_t$ is the driving variable for wage inflation, and

$$\pi_t^w = \kappa_w \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i (mrs_{t+i} - \omega_{t+i}).$$

   (b) Carefully explain the factors generating inefficiencies in this economy. *First, imperfect competition characterizes both the goods market and the labor market in a standard NK model with sticky prices and wages. So this is a key source of inefficiency. Second, sticky prices and stick wages create further distortions – each leads to a dispersion of relative prices or wages, respectively. This dispersion causes households (firms) to purchase an inefficient combination of goods (labor types). Dispersion in relative prices and wages work like negative productivity disturbances in cause more hours of work to be needed to produce a given basket of consumption. Since working generates a disutility, welfare is reduced by price and/or wage dispersion.*

(c) Suppose fiscal taxes and subsidies are used to eliminate the *average* distortions caused by imperfect competition. Can monetary policy eliminate the remaining distortion(s)? Carefully explain your answer. *The key here is the role of the real wage. If the output gap is to be kept at zero (i.e., so output can move with the flex-price/wage output), then the real wage will generally need to adjust to ensure labor market equilibrium in the fasce of shocks. If both prices and wages are sticky, then the real wage will not be able to adjust to match what it would do in the flex-price/wage equilibrium. Monetary policy can stabilize prices (or wages), but then wage inflation (price inflation) and the output gap will move in the face of shocks since the real wage doesn't jump to ensure the output gap remains at zero. With two nominal rigidities, one policy instrument isn't sufficient.*

(d) Discuss the factors that influence the relative weight the policy maker should put on maintaining price inflation and wage inflation at zero, i.e., what determines the weight λ_w in a loss function of the form

$$L_t = \frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda_x x_{t+i}^2 + \lambda_w \left(\pi_{t+i}^w\right)^2 \right],$$

where π_t is price inflation and π_t^w is wage inflation? (Assume L_t comes from a quadratic approximation to welfare.) *As noted in part (b), sticky prices (wages) generate a dispersion of relative prices (wages) that is inefficient. The stickier prices are, the more a given volatility of price inflation generates a dispersion of relative prices and welfare loss. The same is true for wage inflation volatility. So the policy maker should focus more weight on stabilizing the stickier of prices and wages. However, the degree of rigidity is not the only thing that matters – the demand elasticity for goods and labor types is also important. For example, relative price dispersion creates a smaller distortion if household do not respond much to relative price movements – i.e., if their demand for individual goods is relative inelastic. So if wages and prices were equally sticky (say, measured by the Calvo parameter), more weight should put on stabilizing prices if demand facing individual firms is more elasticity than the demand for individual labor types.*

(e) **Optional**: Given the loss function from part (d), write a dynare model file to find the response to a positive productivity shock in a new Keynesian model with sticky prices and sticky wages under the optimal commitment policy. (Hint: you could modify NKM_optc.dyn.) Use the values given in the lecture slides to calibrate the model. Set $\omega_p = \omega_w = 0.75$. Now repeat but set $\omega_w = 0.0001$ (essentially flexible wages). How do your results differ?not use part (d) to solve out for a in the wage equation, I think you can get an express for the equilibrium N. Then you'll end up with two equations in N and a. These are nonlinear, so you don't need to solve them further.

3. In a search and matching model of unemployment, suppose the steady-state value of a job vacancy and the value of a filled job to a firm are given by

$$V^V = -c + \beta \left[qV^J + (1-q)V^V \right]$$

and

$$V^J = \mu x - w + \beta(1-s)V^J,$$

where c is the vacancy posting cost, β is the discount factor, q is the job filling rate, s is the separation rate, x is the output produced by the match, w is the wage, and μ is the firm's markup.

(a) Using the assumption of free entry in vacancy posting, what is the equilibrium value of a job vacancy? *The value of a vacancy is zero under the assumption of free entry.*

(b) Using your result in (a), solve for the value of a filled job as a function of the cost of posting a vacancy, β and the job filling rate. *Explain* why the value of a filled job falls as it becomes easier to fill jobs (i.e., why does V^J fall when q rises?) *With* $V^V = 0$,

$$V^V = -c + \beta \left[qV^J + (1-q)V^V \right]$$

implies

$$V^J = \frac{c}{\beta q}.$$

The value of a filled job must compensate the firm for the cost of filling the job. This is c times the expected number of periods it takes to fill it. As q rises, vacancies are filled more quickly, so the expected cost of filling a job falls. Expressed differently, when jobs become easier to fill (i.e., when q rises), firms post more vacancies and employment hires. This reduces the marginal value of a filled job.

(c) Suppose the value to a worker of being employed rather than unemployed is

$$V^e = w - b + \beta(1-s)V^e,$$

where b is an unemployment benefit. What is the joint surplus to the worker and the firm of being in a match? Explain how the joint surplus is affected by the wage. *From*

$$V^J = \mu x - w + \beta(1-s)V^J = \frac{\mu x - w}{1-\beta(1-s)},$$

and

$$V^e = w - b + \beta(1-s)V^e = \frac{w-b}{1-\beta(1-s)},$$

the joint surplus is $V^{JS} = V^J + V^e$ *or*

$$\begin{aligned} V^{JS} &= \left[\mu x - w + \beta(1-s)V^J\right] + \left[w - b + \beta(1-s)V^e\right] \\ &= \mu x - b + \beta(1-s)V^{JS} \\ &= \frac{\mu x - b}{1-\beta(1-s)}. \end{aligned}$$

The joint surplus is unaffected by the wage – the wage is just a mechanism used to divide the surplus between the worker and the firm.

(d) Suppose the wage is set in Nash bargaining between the worker and the firm, with the worker receiving a share η of the joint surplus. Show that

$$w = (1-\eta)\, b + \eta \mu x.$$

With Nash bargaining an fixed bargaining weights, the wage maximizes

$$(V^e)^\eta \left(V^J\right)^{1-\eta}$$

or

$$\left(\frac{w-b}{1-\beta(1-s)}\right)^\eta \left(\frac{\mu x - w}{1-\beta(1-s)}\right)^{1-\eta}$$

and the first-order condition is

$$
\begin{aligned}
0 \quad = \quad & \frac{\eta}{1-\beta(1-s)}\left(\frac{w-b}{1-\beta(1-s)}\right)^{\eta-1}\left(\frac{\mu x-w}{1-\beta(1-s)}\right)^{1-\eta} \\
& -\frac{1-\eta}{1-\beta(1-s)}\left(\frac{w-b}{1-\beta(1-s)}\right)^{\eta}\left(\frac{\mu x-w}{1-\beta(1-s)}\right)^{-\eta}
\end{aligned}
$$

Simplifying,

$$\eta\left(\frac{w-b}{1-\beta(1-s)}\right)^{-1}\left(\frac{\mu x-w}{1-\beta(1-s)}\right)=(1-\eta)$$

or

$$\eta\left(\mu x-w\right)=\left(1-\eta\right)\left(w-b\right)$$

Solving for w*,*

$$w=\left(1-\eta\right)b+\eta\mu x.$$