Economics 205C
Spring 2017

Problem Set 3: Due in class, Tuesday May 23

1. The NK two-country model can be approximated around a zero steady-state inflatio rate to obtain

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma_0}\right)\left(i_t - \mathrm{E}_t \pi_{h,t+1} - \tilde{\rho}_t\right), \tag{1}$$

where $x_t = y_t - y_t^f$ is the output gap, $\sigma_0 = \sigma\left[1 + \gamma\left(1-\sigma\right)\right]$, and

$$\tilde{\rho}_t \equiv \sigma_0\left(\mathrm{E}_t y_{t+1}^f - y_t^f\right) - \gamma\left(1-\sigma\right)\left(\mathrm{E}_t y_{t+1}^* - y_t^*\right).$$

In the definition of $\tilde{\rho}_t$, y_t^* is foreign income. The domestic flex-price output is defined as

$$y_t^f = \frac{\gamma\left(1-\sigma\right) y_t^* + \left(1+\eta\right) a_t}{\eta + \sigma + \gamma\left(1-\sigma\right)}. \tag{2}$$

Domestic product price inflation is given by

$$\pi_{h,t} = \beta \mathrm{E}_t \pi_{h,t+1} + \bar{\kappa} x_t + u_t, \tag{3}$$

where $\bar{\kappa} = \kappa\left[\eta + \sigma + \gamma\left(1-\sigma\right)\right]$. Assume that

$$a_t = \delta_a a_{t-1} + \varepsilon_{a,t},$$

and

$$u_t = \delta_u u_{t-1} + \varepsilon_{u,t},$$

and that social welfare is given by

$$\frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{h,t+i}^2 + \lambda x_{t+i}^2\right).$$

(a) Assume policy is given by

$$i_t = \phi_\pi \pi_{h,t}.$$

Simulate the model's response to a one standard deviation shock to a_t. (Assume the following values for the parameters: $\sigma = 1.0$, $\beta = 0.99$, $\gamma = 0.5$, $\eta = 3$, $\kappa = 0.05$, $\phi_\pi = 1.5$, $\lambda = \kappa/\theta$, $\theta = 6$, and $\delta_a = \delta_u = 0.9$, $\sigma_a = \sigma_u = 1$.) Plot the responses of the output gap, domestic product price inflation, CPI inflation, the terms of trade, and the nominal exchange rate to this shock. Explain why each of these last three variables responds the way it does. *See nk_ps3_q1.dyn.*

(b) Continuing with the parameter values used in part (a), approximate the loss function by $\sigma_{\pi_h}^2 + \lambda\sigma_x^2$ and calculate the value of this loss function for $\phi_\pi = 1.5$ and for $\phi_\pi = 5$. (**Note**: everyone seems to have set $\sigma_u = 0$ for calculating the loss, so I'll do the same when I report losses.) Which choice leads to the lowest loss? Based on a comparison of the impulse response

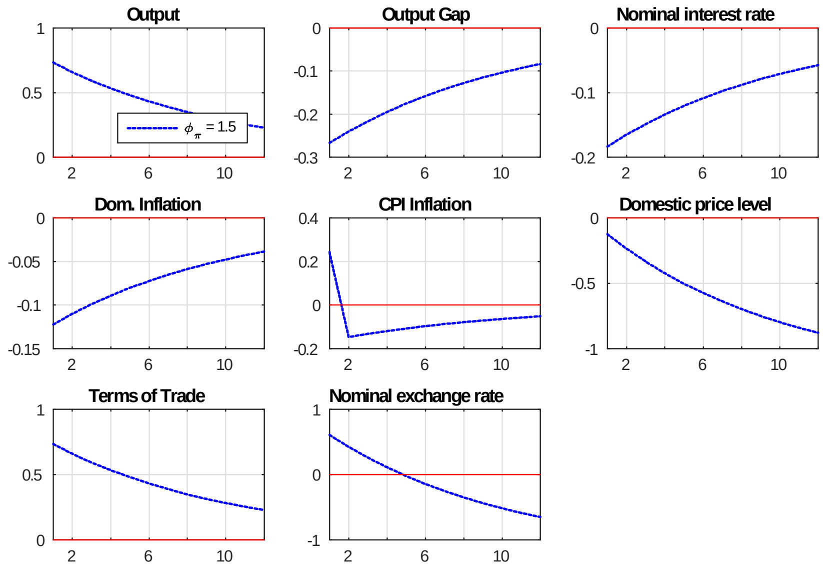

Figure 1: Impulse responses to a_t when policy responds to domestic price inflation and $\phi_\pi = 1.5$.

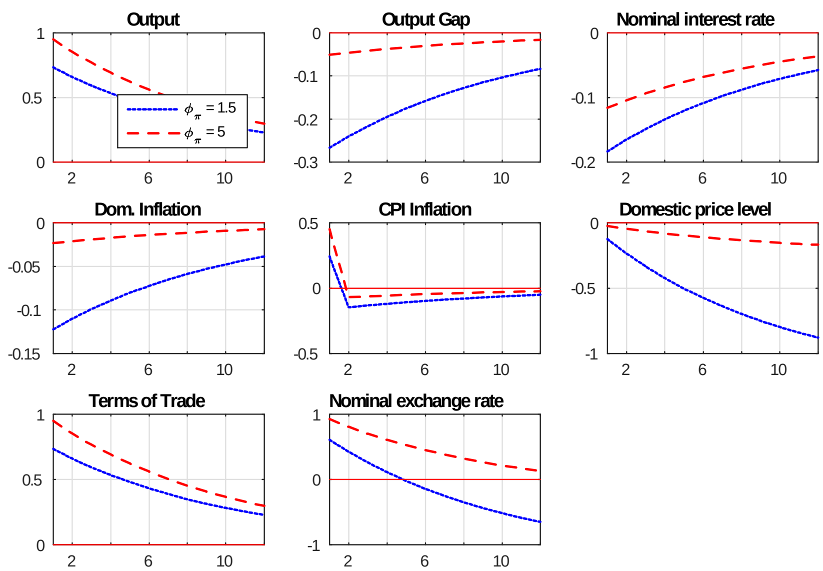

Figure 2: Impulse responses to a_t when policy responds to domestic price inflation and $\phi_\pi = 1.5$ versus $\phi_\pi = 5$.

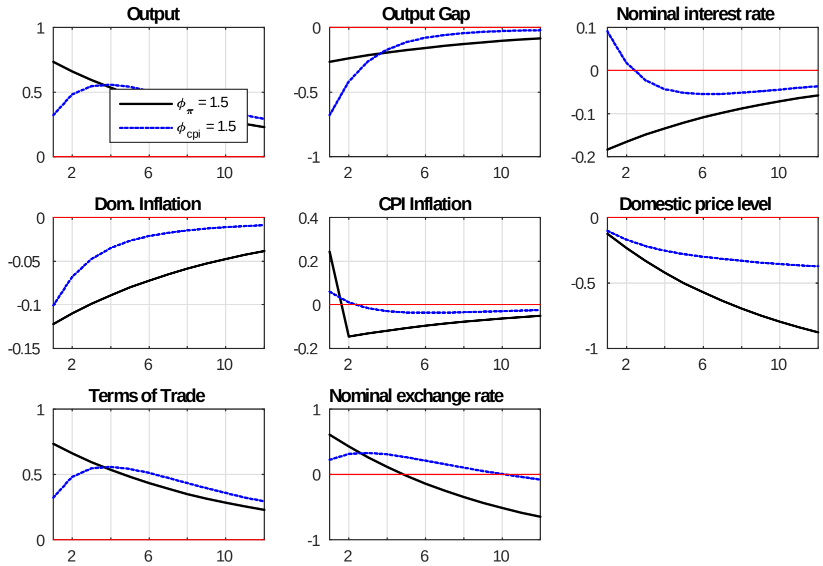

Figure 3: Impulse responses to a_t when policy responds to consumer price inflation and $\phi_\pi = 1.5$.

functions, can you explain why one policy does better than the other? *The loss for* $\phi_\pi = 1.5$ *is* 0.018 *and* 0.0029 *when* $\phi_\pi = 5.0$. *The stronger response to inflation leads to more stable domestic price inflation and a more stable output gap. Recall that optimal policy would completely stabilize domestic inflation in the face of a productivity shock. The positive shock to aggregate productivity increases the flex-price output level. By reacting strongly (i.e., when* ϕ_π *is larger) to the downward pressure on domestic price inflation generated by the rise in flex-price output, monetary policy ensures the output gap remains smaller.*

(c) Repeat part (b) but now assume policy is given by

$$i_t = \phi_\pi \pi_t,$$

where π_t is CPI inflation. Set $\phi_\pi = 1.5$. Evaluate the loss function with this rule. , and discuss how the responses differ from those you found in part (a) when $\phi = 1.5$. *Compare the figures. The loss under the consumer price inflation rule, the loss is* 0.0269.

(d) Now suppose the policy rule is

$$i_t = 1.5\pi_{h,t} + 10e_t,$$

where e_t is the nominal exchange rate. Evaluate the loss function with this rule. How does it compare to the rules you evaluated in parts (a) and (c)? Which does better? Can you explain why?*See nk_ps3_q1.dyn.. The loss equals* 0.0550. *The productivity shock causes a nominal exchange rate depreciation under the inflation targeting rules as the nominal interest rate is cut to ensure demand rises with the rise in output. To prevent this depreciation, the policy calls for not cutting the nominal interest rate as much. Thus, aggregate demand rises by less than the increase in aggregate supply, producing a negative output gap which lowers domestic price inflation.*

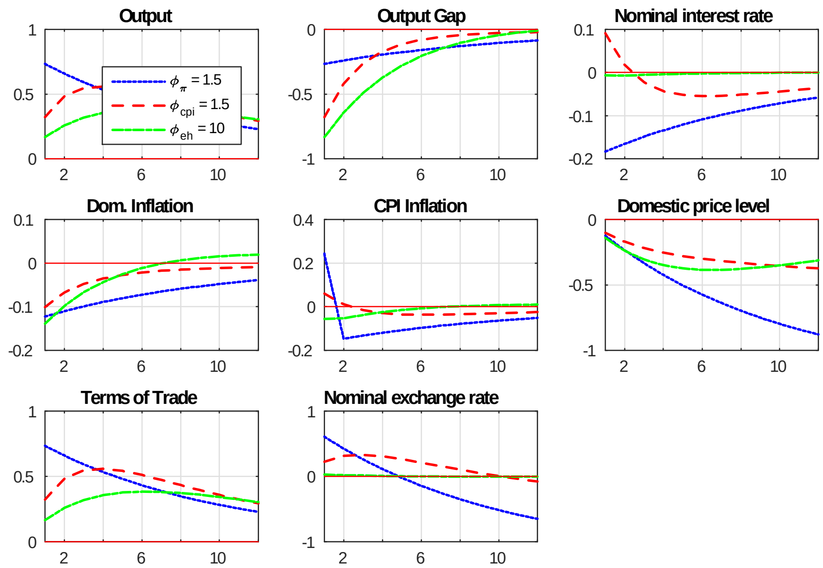

Figure 4: Exchange rate targeting version inflation targeting

2. Consider a firm that can invest in one of two projects. Project $i = 1, 2$ yields a gross rate of return of $R - x_i$ with probability $1/2$ and $R + x_i$ with probability $1/2$. Assume $x_2 > x_1$ so project 2 is a riskier project. The firm borrows L to undertake the project and has collateral C. The lender's opportunity cost of funds (the rate of return it can earn if it does not lend to the firm) is r. Both the firm and the lender are risk neutral. Assume the firm defaults when $R - x_i$ occurs, in which case the lender gets $R - x_i + C$.

   (a) Suppose the lender can, without cost, monitor which project the firm chooses. What interest rate will the lender charge the firm if the firm picks project 1? What interest rate would it charge if the firm picks project 2? (Hint: for either project, the expected rate of return to the lender must equal r.) *On project i, the lender's expected return is*

$$\frac{1}{2}\left(1 + r_i^L\right) L + \frac{1}{2}\left[R - x_i + C\right]$$

   and this must equal $(1 + r) L$. *Hence,*

$$\frac{1}{2}\left(1 + r_i^L\right) L + \frac{1}{2}\left[R - x_i + C\right] = (1 + r) L$$

   implies

$$\left(r_i^L - r\right) L = \left[(1 + r) L - C - R + x_i\right],$$

   or

$$\left(r_i^L - r\right) = \frac{\left[(1 + r) L + x_i - C - R\right]}{L},$$

   so the lender charges a higher interest rate on the loan for the riskier project. (Since the lender receives more in the good state, and the average (expected) return to the lender must equal $1 + r$, it follows that $r_i^L - r > 0$.

(b) Now suppose the firm chooses which project to undertake after it receives the loan, and the lender cannot observe which project is undertaken. What interest rate will be the bank charge on loans? Can low-risk projects get funding? Explain. *The firm's expected return is higher on the high risk project 2. So the lender has to assume only type 2 projects will be undertaken. The loan rate will therefore be* $r_2^L > r_1^L$ *and only high-risk projects are undertaken.*

3. Suppose a bank's balance sheet is

$$d_t + n_t = a + b,$$

where d represents the bank's deposit liabilities, n is its capital, a are private sector assets, and b are holdings of government bonds. The bank wishes to maximize its net income defined as

$$n_{t+1} = R_{a,t}a_t + R_{b,t}b_t - R_{d,t}d_t$$

where $R_{a,t}$ and $R_{b,t}$ are the gross returns (i.e., one plus the rate of return) on assets and bonds, and $R_{d,t}$ is the rate paid on deposits.

(a) Assume the banking sector is perfectly competitive and frictionless. Show that the first-order conditions for the bank's problem of maximizing n_{t+1} subject to its balance sheet constraint implies $R_{a,t} = R_{b,t} = R_{d,t}$. *Using the balance sheet constraint o eliinate* d_t*, the problem becomes one of maximizing*

$$R_{a,t}a_t + R_{b,t}b_t - R_{d,t}\left(a_t + b_t - n_t\right)$$

and the first order conditions are $R_{a,t} - R_{d,t} = 0$ *and* $R_{b,t} - R_{d,t} = 0$*. More specifically, the decision problem is unbounded if these spreads are not equal to zero.*

(b) Now assume the bank owners can divert a fraction θ_a of their asset holdings and a fraction $\theta_b < \theta_a$ of their bond holdings for their own use. They will now face an incentive constraint of the form

$$n_{t+1} \geq \theta\left(a_t + \omega b_t\right).$$

If this does not hold, the bank owers have an incentive to run off with the funds they can divert. Now solve for the bank's problem of maximizing n_{t+1} subject to the balance sheet constraint and the incentive constraint. Write down the first-orer conditions for this problem. *The bank's problem can now be written in terms of the Lagrangian*

$$\begin{aligned} L &= \left(R_{a,t} - R_t\right)a_t + \left(R_{b,t} - R_t\right)b_t + R_{d,t}n_t \\ &\quad + \lambda\left[\left(R_{a,t} - R_t\right)a_t + \left(R_{b,t} - R_t\right)b_t + R_{d,t}n_t - \theta\left(a_t + \omega b_t\right)\right] \end{aligned}$$

and the FOCs are

$$(1+\lambda)\left(R_{a,t} - R_{d,t}\right) - \lambda\theta = 0$$

$$(1+\lambda)\left(R_{b,t} - R_{d,t}\right) - \lambda\theta\omega = 0$$

or

$$R_{a,t} - R_{d,t} = \frac{\lambda}{1+\lambda}\theta$$

$$R_{b,t} - R_{d,t} = \frac{\lambda}{1+\lambda}\theta\omega$$

(c) Show under what conditions $R_{a,t} > R_{b,t} > R_{d,t}$. *From part (b),*

$$R_{a,t} - R_{b,t} = \frac{\lambda}{1+\lambda}\theta(1-\omega)$$

$$R_{b,t} - R_{d,t} = \frac{\lambda}{1+\lambda}\theta\omega$$

so $R_{a,t} > R_{b,t} > R_{d,t}$ *requires* $\lambda > 0$ *so that the incentive constraint is binding, and* $\omega < 1$.

4. Consider a contract between a borrower B and a lender L. Both are risk neutral and the lender's required expected rate of return is r. The borrower has funds S to invest; the project requires an investment of $\$1 > S$ The project yields a payout of $(1 + r^p + \varepsilon)$, where $r^P > r$ and ε is a mean zero return shock uniformly distributed over the interval $[-\bar{\varepsilon}\ \bar{\varepsilon}]$.

   (a) Suppose ε is realized and observed by both borrower and lender before investment in the project takes place. Find an expression for ε, call it ε^*, such that all projects with $\varepsilon \geq \varepsilon^*$ would be funded in an efficient equilibirum. Does ε^* depend on S? Explain. *Efficiency requires that the return exceed* r, *so* ε^* *is such that* $r^p + \varepsilon^* = r \Rightarrow \varepsilon^* = r - r^p < 0$. *Any project with* $\varepsilon < \varepsilon^*$ *would not be funded. The profitability of the project is independent of who funds it, so* ε^* *is independent of* S.

   (b) Now assume ε is realized after investment in the project takes place and that the borrower observes the realization of ε but the lender must pay c to observe ε. The optimal lending contract must maximize the borrower's expected return subject to two constraints: the lender's expected return must equal r and the borrower must have an incentive to truthfully reveal the realized value of ε. *Explain in words* why the optimal contract will be characterized by a value ε' such that if $\varepsilon \geq \varepsilon'$, the borrower pays a fixed amount K per dollar borrowed to the lender, where K is independent of ε, and if $\varepsilon < \varepsilon'$ the lender takes over the entire project and receives $1 + r^p + \varepsilon - c$, where c is the fixed cost for the lender of assessing the value of the project. *Since the lender does not observe project outcomes except by paying a cost, the optimal contract would minimize costs consistent with ensuring the lender's expected return is* r. *Clearly the lender's costs are minimized if projects are never monitored, but then all firms would never truthfully report their outcomes. If repayment* K *depends on the borrower's report on* ε, *then the borrower will always have an incentive to report the lowest value of* ε, *forcing the lender to pay the cost of auditing. Consider the following simple example. There are three values of* ε *each with probability one-third. Consider payments of* K_1, K_2 *and* K_3 *if* $\varepsilon_1 < \varepsilon_2 < \varepsilon_3$ *occurs. The return to the lender must equal* r *so*

$$R_A \equiv \frac{1}{3}(K_1 + K_2 + K_3) - c = r$$

   since the cost must be incurred in every case if K_i *depends on* ε_i. *Now consider an alternative in which the lender only verifies outcome if* ε_1 *is reported. Since a report of* ε_2 *or* ε_3 *is not verified, the borrower would always report the value that leads to the lowest payment, so the lender will receive the same amount which ever occurs. Call this* K. *The return to the lender is*

$$R_B \equiv \frac{1}{3}(K_1 - c) + \frac{2}{3}K = r$$

Now

$$R_B - R_A = \frac{2}{3}K - \frac{1}{3}(K_2 + K_3) + \frac{2}{3}c = 0$$

or

$$K = \frac{1}{2}(K_2 + K_3) - c < \frac{1}{2}(K_2 + K_3)$$

so contract B yields the lender the same return as contract A but lowers the expected payment of the borrower:

$$\frac{1}{3}K_1 + \frac{2}{3}K < \frac{1}{3}K_1 + \frac{2}{3}\frac{1}{2}(K_2 + K_3) < \frac{1}{3}(K_1 + K_2 + K_3).$$

So a contract that makes repayment independent of ε except for ε_1 is reported dominates the one in which K_i depends on ε_i for all i.

(c) Find an expression for ε' as a function of K and show it is decreasing in S. Explain. *The value of ε' will be such that the borrower has just enough to repay K:*

$$(1 + r^p) + \varepsilon' = KB \Rightarrow \varepsilon' = K(1 - S) - (1 + r^p).$$

The higher S is, the less the firm had to borrow, so the lower is the "break even" ε that allows the firm to repay the loan.

(d) Use your result in (c) to express the borrower's expected payout Π^B as a function of ε'.

$$\begin{aligned}
\mathrm{E}\left(\Pi^B\right) &= \int_{\varepsilon'}^{\bar{\varepsilon}} \left[1 + r^p + \varepsilon - K(1 - S)\right]\left(\frac{1}{2\bar{\varepsilon}}\right) d\varepsilon \\
&= \left(\frac{1}{2\bar{\varepsilon}}\right)\int_{\varepsilon'}^{\bar{\varepsilon}} \varepsilon d\varepsilon + \left[1 - F(\varepsilon')\right]\left[1 + r^p - K(1 - S)\right] \\
&= \left(\frac{1}{4\bar{\varepsilon}}\right)\left[\bar{\varepsilon}^2 - (\varepsilon')^2\right] - \left[1 - F(\varepsilon')\right]\varepsilon' \\
&= \left(\frac{1}{4\bar{\varepsilon}}\right)\left[\bar{\varepsilon}^2 - (\varepsilon')^2\right] - \left(\frac{1}{2\bar{\varepsilon}}\right)(\bar{\varepsilon} - \varepsilon')\varepsilon'.
\end{aligned}$$

(e) What is the lender's expected payout as a function of ε'? *Expected payout to lender is*

$$
\begin{aligned}
\mathrm{E}\left(\Pi^L\right) &= [1 - F(\varepsilon')]\, KB + \left(\frac{1}{2\bar{\varepsilon}}\right) \int_{-\bar{\varepsilon}}^{\varepsilon'} (1 + r^p + \varepsilon - c)\, d\varepsilon \\
&= [1 - F(\varepsilon')]\, K(1 - S) + \left(\frac{\varepsilon' + \bar{\varepsilon}}{2\bar{\varepsilon}}\right)(1 + r^p - c) \\
&\quad + \left(\frac{1}{2\bar{\varepsilon}}\right) \int_{-\bar{\varepsilon}}^{\varepsilon'} \varepsilon d\varepsilon \\
&= [1 - F(\varepsilon')]\,(\varepsilon' + 1 + r^p) + \left(\frac{\varepsilon' + \bar{\varepsilon}}{2\bar{\varepsilon}}\right)(1 + r^p - c) \\
&\quad + \left(\frac{1}{4\bar{\varepsilon}}\right)\left[(\varepsilon')^2 - \bar{\varepsilon}^2\right] \\
&= \left(\frac{\bar{\varepsilon} - \varepsilon'}{2\bar{\varepsilon}}\right)(\varepsilon' + 1 + r^p) + \left(\frac{\varepsilon' + \bar{\varepsilon}}{2\bar{\varepsilon}}\right)(1 + r^p - c) \\
&\quad + \left(\frac{1}{4\bar{\varepsilon}}\right)\left[(\varepsilon')^2 - \bar{\varepsilon}^2\right] \\
&= (1 + r^p) + \left(\frac{\bar{\varepsilon} - \varepsilon'}{2\bar{\varepsilon}}\right)\varepsilon' - \left(\frac{\varepsilon' + \bar{\varepsilon}}{2\bar{\varepsilon}}\right) c \\
&\quad + \left(\frac{1}{4\bar{\varepsilon}}\right)\left[(\varepsilon')^2 - \bar{\varepsilon}^2\right].
\end{aligned}
$$