Economics 205C
Spring 2017

Problem Set 3: Due in class, Tuesday May 23

1. The NK two-country model can be approximated around a zero steady-state inflatio rate to obtain

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma_0}\right)(i_t - \mathrm{E}_t \pi_{h,t+1} - \tilde{\rho}_t), \tag{1}$$

where $x_t = y_t - y_t^f$ is the output gap, $\sigma_0 = \sigma[1 + \gamma(1-\sigma)]$, and

$$\tilde{\rho}_t \equiv \sigma_0 \left(\mathrm{E}_t y_{t+1}^f - y_t^f\right) - \gamma(1-\sigma)\left(\mathrm{E}_t y_{t+1}^* - y_t^*\right).$$

In the definition of $\tilde{\rho}_t$, y_t^* is foreign income. The domestic flex-price output is defined as

$$y_t^f = \frac{\gamma(1-\sigma) y_t^* + (1+\eta) a_t}{\eta + \sigma + \gamma(1-\sigma)}. \tag{2}$$

Domestic product price inflation is given by

$$\pi_{h,t} = \beta \mathrm{E}_t \pi_{h,t+1} + \bar{\kappa} x_t + u_t, \tag{3}$$

where $\bar{\kappa} = \kappa[\eta + \sigma + \gamma(1-\sigma)]$. Assume nd that

$$a_t = \delta_a a_{t-1} + \varepsilon_{a,t},$$

and

$$u_t = \delta_u u_{t-1} + \varepsilon_{u,t},$$

and that social welfare is given by

$$\frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{h,t+i}^2 + \lambda x_{t+i}^2\right).$$

(a) Assume policy is given by

$$i_t = \phi_\pi \pi_{h,t}.$$

Simulate the model's response to a one standard deviation shock to a_t. (Assume the following values for the parameters: $\sigma = 1.0$, $\beta = 0.99$, $\gamma = 0.5$, $\eta = 3$, $\kappa = 0.05$, $\phi_\pi = 1.5$, $\lambda = \kappa/\theta$, $\theta = 6$, and $\delta_a = \delta_u = 0.9$, $\sigma_a = \sigma_u = 1$.) Plot the responses of the output gap, domestic product price inflation, CPI inflation, the terms of trade, and the nominal exchange rate to this shock. Explain why each of these last three variables responds the way it does.

(b) Continuing with the parameter values used in part (a), approximate the loss function by $\sigma_{\pi_h}^2 + \lambda \sigma_x^2$ and calculate the value of this loss function for $\phi_\pi = 1.5$ and for $\phi_\pi = 5$. Which choice leads to the lowest loss? Based on a comparison of the impulse response functions, can you explain why one policy does better than the other?

(c) Repeat part (b) but now assume policy is given by

$$i_t = \phi_\pi \pi_t,$$

where π_t is CPI inflation. Set $\phi_\pi = 1.5$. Evaluate the loss function with this rule. , and discuss how the responses differ from those you found in part (a) when $\phi = 1.5$.

(d) Now suppose the policy rule is

$$i_t = 1.5\pi_{h,t} + 10e_t,$$

where e_t is the nominal exchange rate. Evaluate the loss function with this rule. How does it compare to the rules you evaluated in parts (a) and (c)? Which does better? Can you explain why?

2. Consider a firm that can invest in one of two projects. Project $i = 1, 2$ yields a gross rate of return of $R - x_i$ with probability 1/2 and $R + x_i$ with probability 1/2. Assume $x_2 > x_1$ so project 2 is a riskier project. The firm borrows L to undertake the project and has collateral C. The lender's opportunity cost of funds (the rate of return it can earn if it does not lend to the firm) is r. Both the firm and the lender are risk neutral. Assume the firm defaults when $R - x_i$ occurs, in which case the lender gets $R - x_i + C$.

   (a) Suppose the lender can, without cost, monitor which project the firm chooses. What interest rate will the lender charge the firm if the firm picks project 1? What interest rate would it charge if the firm picks project 2? (Hint: for either project, the expected rate of return to the lender must equal r.)

   (b) Now suppose the firm chooses which project to undertake after it receives the loan and the lender cannot observe which project is undertaken. What interest rate will be the bank charge on loans? Can good, i.e., low risk, projects get funding? Explain.

3. Suppose a bank's balance sheet is

$$d_t + n_t = a + b,$$

where d represents the bank's deposit liabilities, n is its capital, a are private sector assets, and b are holdings of government bonds. The bank wishes to maximize its net income defined as

$$n_{t+1} = R_{a,t}a_t + R_{b,t}b_t - R_{d,t}d_t$$

where $R_{a,t}$ and $R_{b,t}$ are the gross returns (i.e., one plus the rate of return) on assets and bonds, and $R_{d,t}$ is the rate paid on deposits.

   (a) Assume the banking sector is perfectly competitive and frictionless. Show that the first-order conditions for the bank's problem of maximizing n_{t+1} subject to its balance sheet constraint implies $R_{a,t} = R_{b,t} = R_{d,t}$.

   (b) Now assume the bank owners can divert a fraction θ_a of their asset holdings and a fraction $\theta_b < \theta_a$ of their bond holdings for their own use. They will now face an incentive constraint of the form

$$n_{t+1} \geq \theta \left(a_t + \omega b_t\right).$$

   If this does not hold, the bank owers have an incentive to run off with the funds they can divert. Now solve for the bank's problem of maximizing n_{t+1} subject to the balance sheet constraint and the incentive constraint. Write down the first-orer conditions for this problem.

(c) Show under what conditions $R_{a,t} > R_{b,t} > R_{d,t}$.

4. Consider a contract between a borrower B and a lender L. Both are risk neutral and the lender's required expected rate of return is r. The borrower has funds S to invest; the project requires an investment of $\$1 > S$ The project yields a payout of $(1 + r^p + \varepsilon)$, where $r^P > r$ and ε is a mean zero return shock uniformly distributed over the interval $[-\bar{\varepsilon}\ \bar{\varepsilon}]$.

   (a) Suppose ε is realized and observed by both borrower and lender before investment in the project takes place. Find an expression for ε, call it ε^*, such that all projects with $\varepsilon \geq \varepsilon^*$ would be funded in an efficient equilibirum. Does ε^* depend on S? Explain.

   (b) Now assume ε is realized after investment in the project takes place. The borrower observes the realization of ε but the lender must pay c to observe ε. The optimal lending contract maximizes the borrower's expected return subject to ensuring the lender's expected return is r and the borrower truthfully reveals the realized ε. *Explain in words* why the optimal contract will be characterized by a ε' such that if $\varepsilon \geq \varepsilon'$, the borrower pays a fixed amount K per dollar borrowed to the lender, where K is independent of ε, and if $\varepsilon < \varepsilon'$ the lender takes over the entire project and receives $1 + r^p + \varepsilon - c$.

   (c) Find an expression for ε' as a function of K and show it is decreasing in S. *Explain.*

   (d) Use your result in (c) to express the borrower's expected payout Π^B as a function of ε'.

   (e) What is the lender's expected payout as a function of ε'?