Economics 205C
Spring 2017

Problem Set 2: Answers

1. Suppose the central bank acts to minimize

$$\left(\frac{1}{2}\right)\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left(\pi_{t+i}^2+\lambda_x x_{t+i}^2\right).$$

The structure of the economy is given by

$$\pi_t=(1-\phi)\,\beta\mathrm{E}_t\pi_{t+1}+\phi\pi_{t-1}+\kappa x_t+e_t \tag{1}$$

$$x_t=\mathrm{E}_t x_{t+1}-\left(\frac{1}{\sigma}\right)\left(i_t-\mathrm{E}_t\pi_{t+1}-r_t\right), \tag{2}$$

where e and r are exogenous stochastic shocks. The parameter ϕ is bounded between zero and one. Let ψ_t denote the Lagrangian multiplier on constraint (1) and let ξ_t be the multipler on (2).

(a) Derive the first order conditions for the fully optimal policy of the central bank under commitment. Eliminate any Lagrangian multipliers and express the first order condition as a targeting criterion involving inflation and the output gap. *Under commitment, the decision problem is*

$$\begin{aligned}&\min\mathrm{E}_t\sum_{j=0}^{\infty}\beta^j\left\{\frac{1}{2}\left(\pi_{t+j}^2+\lambda x_{t+j}^2\right)\right.\\&+\psi_{t+j}\left(\pi_{t+j}-(1-\phi)\,\beta\pi_{t+1+j}-\phi\pi_{t+j-1}-\kappa x_{t+j}-e_{t+j}\right)\\&\left.+\xi_{t+j}\left[x_{t+j}-\mathrm{E}_t x_{t+1+j}+\left(\frac{1}{\sigma}\right)\left(i_{t+j}-\pi_{t+1+j}-r_{t+j}\right)\right]\right\}.\end{aligned}$$

The first order conditions under commitment are

$$\pi_t+\psi_t-\beta\phi\mathrm{E}_t\psi_{t+1}=0$$

$$\mathrm{E}_t\left[\pi_{t+j}+\psi_{t+j}-(1-\phi)\,\psi_{t+j-1}-\beta\phi\psi_{t+j+1}-\left(\frac{1}{\sigma}\right)\beta^{-1}\xi_{t+j-1}\right]=0;\ j>0$$

$$\lambda_x x_t-\psi_t\kappa+\xi_t=0$$

$$\mathrm{E}_t\left(\lambda_x x_{t+j}-\psi_{t+j}\kappa+\xi_{t+j}-\beta^{-1}\xi_{t+j-1}\right)=0;\ j\geq 0$$

$$\left(\frac{1}{\sigma}\right)\mathrm{E}_t\xi_{t+j}=0;\ j\geq 0.$$

These become

$$\kappa\pi_t+\lambda_x x_t-\beta\phi\lambda_x\mathrm{E}_t x_{t+1}=0$$

$$\mathrm{E}_t\left[\kappa\pi_{t+j}+\lambda_x x_{t+j}-(1-\phi)\lambda_x x_{t+j-1}-\beta\phi\lambda_x x_{t+j+1}\right]=0,\ j>0$$

(b) Derive the first order conditions for the optimal optimal commitment policy from a timeless perspective. Eliminate any Lagrangian multipliers and express the first order condition as a targeting rule. *From the results in part (a),*

$$\mathrm{E}_t\left[\pi_{t+j}+\psi_{t+j}-(1-\phi)\psi_{t+j-1}-\beta\phi\psi_{t+j+1}\right]=0;\ j\geq 0$$

$$\mathrm{E}_t\left(\lambda_x x_{t+j}-\psi_{t+j}\kappa\right)=0$$

These become

$$\kappa\pi_t+\lambda_x x_t-(1-\phi)\lambda_x x_{t-1}-\beta\phi\lambda_x\mathrm{E}_t x_{t+1}=0$$

or

$$\kappa\pi_t+(1-\phi)\lambda_x\left(x_t-x_{t-1}\right)-\beta\phi\lambda_x\left(\mathrm{E}_t x_{t+1}-\beta^{-1}x_t\right)=0 \quad (3)$$

(c) Explain why optimal policy in (b) is both forward looking and backward looking. *Optimal commitment is backward-looking for the same reasons discussed earlier. Policy is now forward-looking because of the presence of lagged inflation in the inflation adjustment equation. The central bank's choice of inflation today will affect future inflation through this lagged adjustment. So the policy maker needs to take into account how decisions today affect the future. This leads to the presence of the expected future output gap in the optimal targeting criterion (3).*

2. Write a Dynare program to solve the following model:

$$x_t=\mathrm{E}_t x_{t+1}-\left(\frac{1}{\sigma}\right)\left(i_t-\mathrm{E}_t\pi_{t+1}-r_t^n\right)$$

$$\pi_t=\beta\mathrm{E}_t\pi_{t+1}+\kappa x_t+u_t$$

$$r_t^n=\rho_r r_{t-1}^n+e_{r,t}$$

$$u_t=\rho_u u_{t-1}+e_{u,t}$$

where $e_{z,t}$ is a mean zero, white noise process for $z=r,u,v$. Set $\sigma=2$, $\beta=0.99$, $\kappa=0.15$, $\phi_\pi=1.5$, $\phi_x=0$, $\rho_r=0.9$, $\rho_u=0.9$, and $\rho_v=0.5$. Set the standard deviations of $e_{r,t}$ and $e_{u,t}$ to 0.01. Assume the central bank wishes to minimize a loss function given by

$$L=\left(\frac{1}{1-\beta}\right)\left(\sigma_\pi^2+\lambda\sigma_x^2\right).$$

Set $\lambda=0.25$.

(a) Solve the model under optimal discretion and under optimal commitment. Calculate L in each case. *See nk_ps2_q2.dyn.*

(b) Now assume policy is described by the following instrument rule:

$$i_t = r_t^n + \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi \pi_t + \phi_x x_t + v_t\right).$$

The new parameter ρ_i captures inertia in the central bank's policy rule. When $\rho_i = 0$ we have the basic model analyzed previously. Solve the model for $\rho_i = 0, 0.25, 0.5,$ and 0.75. Set $\phi_\pi = 1.5$ and $\phi_x = 0.125$. For each value of ρ_i, record the value of the loss function. (If you modify the code used for part (a), remember to remove the planner_objective statement that the discretionary_policy and ramsey_policy commands require.) *See nk_ps2_q2b.dyn.*

(c) How does loss vary with ρ_i? Does inertia in policy improve over discretion? If so, explain why? (Hint: it might be useful set ρ_r and ρ_u equal to zero and compare the impulse responses for different ρ_i to those obtained under optimal discretion and optimal commitment.) *Recall that under commitment, policy introduces inertia so that expectations of future inflation move in a way that helps stabilize the economy and produce a better trade-off between inflation and the output gap. Commitment also ends help stabilizing the price level. As ρ_i increases, it adds inertia to the responses and approaches the responses displayed under optimal commitment.*

3. Consider the following model for the output gap and inflation at the zero lower bound:

$$x_t = \mathrm{E}_t x_{t+1} + \sigma^{-1}\left(\mathrm{E}_t \pi_{t+1} + r_t\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + u_t,$$

where u_t is a mean-zero exogenous stochastic disturbance and r_t is an exogenous stochastic disturbances with mean ρ (ρ is the steady-state real interest rate). Assume initially $r_t = r^{zlb} < 0$ such that $i_t = 0$. r_t follows a Markov process such that with probability q, $r_{t+1} = r^{zlb}$ and with probability $1-q$, $r_{t+1} = \bar{r} > 0$. Once $r_{t+i} = \bar{r}$, it remains at $\bar{r}$ and i_{t+i} is set to ensure $x_{t+i} = \pi_{t+i} = 0$.

(a) Assume that $u_t \equiv 0$ for all t. If $r_t = r^{zlb}$, solve for equilibrium values of x_t and π_t as functions of q. *This was solved in class, so the answers here will be brief. Use the assumptions of the problem to solve for the expected output gap and inflation:*

$$\mathrm{E}_t x_{t+1} = q x^Z + (1-q) \times 0 = q x^Z$$

$$\mathrm{E}_t \pi_{t+1} = q \pi^Z + (1-q) \times 0 = q \pi^Z$$

where x^Z and π^Z are the equilibrium values at the zero interest rate. Hence, the two equilbirum conditions become,

$$x^Z = q x^Z + \sigma^{-1}\left(q \pi^Z + r_t\right)$$

$$\pi^Z = \beta q \pi^Z + \kappa x^Z.$$

Write the system as

$$\begin{bmatrix} 1-q & -\frac{q}{\sigma} \\ -\kappa & 1-\beta q \end{bmatrix} \begin{bmatrix} x^Z \\ \pi^Z \end{bmatrix} = \begin{bmatrix} \frac{1}{\sigma} r^{zlb} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x^Z \\ \pi^Z \end{bmatrix} = \left(\frac{1}{A}\right) \begin{bmatrix} (1-q\beta)\, r^{zlb} \\ \kappa r^{zlb} \end{bmatrix} \tag{4}$$

and

$$A \equiv \sigma(1-q)(1-\beta q) - \kappa q$$

(b) How does an increase in q affect x_t and π_t. Explain. *Discussed in class. As long $A > 0$ a rise in q,corresponding to a belief the duration of the ZLB period has risen. This lowers expected future output and inflation. The fall in the later immediately works to lower current inflation. The fall in expected inflation also raises the real interest rate with the nominal rate fixed at zero. This rise in the real rate and fall in expected future output both act to lower current aggregate demand and the output gap. The fall in the later then further reduces current inflation.*

(c) If π_t is interpreted as inflation relative to target (2% in the U.S.), one could argue that from 2009 to 2016 the U.S. was characterized by $i_t = 0$, $x_t < 0$, and $\pi_t = 0$. Is such an outcome consistent with your results in part (a)? If $u_t \neq 0$, are there values of the cost shock that would be consistent with $x_t < 0$ and $\pi_t = 0$? Does it require a positive or negative cost shock? Explain. *The results are not consistent with the findings in part (a) as (4) shows the x^Z and π^Z should have the same sign. If we added back in the cost shock, a positive inflation shock could explain why inflation is zero (or equal to the target) while the output gap is negative. If you think about the simple graphical analysis we looked at in class, a positive cost shock would shift the inflation relationship upwards.*

4. Compare two economies that are identical to the economy in the previous question except that one has more flexible prices, so κ is larger in the economy with more flexible prices. How do the output gaps and inflation rates you found in part (a) compare in these two economies. Which one has the lower output gap at the zero lower bound? Explain. *From (4), a rise in κ reduces A and so increase $1/A$. This makes x^Z unambigiously more negative (since $\bar{r} + r^{zlb} < 0$). Since*

$$\pi^Z = \frac{\kappa(\bar{r} + r^{zlb})}{A},$$

the numerator rises and the denominator falls, so inflation becomes more negative also. A larger κ makes inflation more sensitive to the output gap, so when aggregate demand falls due to the demand shock that is not offset by an interest rate cut, this causes a larger fall in inflation in the economy with more flexible prices. Since the shock is persistent, expected future inflation also falls more, and this increases the real interest rate which further depresses the output gap (and inflation, and so on).

5. Forward guidance (Based on Carlstrom, Furest, and Paustrian 2012). Suppose the economy is represented by a simple NK model consisting of a standard consumption Euler condition and NK Phillips curve:

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

where r_t is an exogenous stochastic disturbance (the only disturbance in the model). Assume from t until $t+N$, $r_{t+i} < 0$ such that the economy is at the ZLB. From $t+N+1$ on, $r_t = \bar{r} > 0$. The central bank credibly commits to keeping the nominal interest rate at zero until $t+N+M$ where $M \geq 0$. For $t+N+M+k$, $k \geq 1$, the monetary authority follows a targeting rule of the form $\pi_t + \psi x_t = 0$. For calibration purposes, set $\sigma = 1$, $\beta = 0.99$, and $\kappa = 0.08$. Set $N = 3$. From t until $t+N$, $r = -0.02$, while $\bar{r} = 0.03$.

(a) Using Matlab (or Excel for that matter), plot the equilibrium for x_{t+i} and π_{t+i}, $i = 0, 1, ..., N+M+3$ for $M = 0, 1, 2$. *See fg_ps2_q6.m.*

(b) Is forward guidence effective in boosting output at the zero lower bound? Briefly explain what is going on. *Yes, as we discussed in class.*