Economics 205C
Spring 2017

Problem Set 2: Due in class, Tuesday May 2

1. Suppose the central bank acts to minimize

$$\left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda_x x_{t+i}^2\right).$$

The structure of the economy is given by

$$\pi_t = (1-\phi)\,\beta \mathrm{E}_t \pi_{t+1} + \phi \pi_{t-1} + \kappa x_t + e_t \quad (1)$$

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t\right), \quad (2)$$

where e and r are exogenous stochastic shocks. The parameter ϕ is bounded between zero and one. Let ψ_t denote the Lagrangian multiplier on constraint (1) and let ξ_t be the multipler on (2).

   (a) Derive the first order conditions for the fully optimal policy of the central bank under commitment. Eliminate any Lagrangian multipliers and express the first order condition as a targeting criterion involving inflation and the output gap.
   (b) Derive the first order conditions for the optimal optimal commitment policy from a timeless perspective. Eliminate any Lagrangian multipliers and express the first order condition as a targeting rule.
   (c) Explain why optimal policy in (b) is both forward looking and backward looking.

2. Write a Dynare program to solve the following model:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + u_t$$

$$r_t^n = \rho_r r_{t-1}^n + e_{r,t}$$

$$u_t = \rho_u u_{t-1} + e_{u,t}$$

where $e_{z,t}$ is a mean zero, white noise process for $z = r, u, v$. Set $\sigma = 2$, $\beta = 0.99$, $\kappa = 0.15$, $\phi_\pi = 1.5$, $\phi_x = 0$, $\rho_r = 0.9$, $\rho_u = 0.9$, and $\rho_v = 0.5$. Set the standard deviations of $e_{r,t}$ and $e_{u,t}$ to 0.01. Assume the central bank wishes to minimize a loss function given by

$$L = \left(\frac{1}{1-\beta}\right)\left(\sigma_\pi^2 + \lambda \sigma_x^2\right).$$

Set $\lambda = 0.25$.

(a) Solve the model under optimal discretion and under optimal commitment. Calculate L in each case. (See Slides_MatlabDynare_2017.pdf in the Matlab folder under files on Canvas.)

(b) Now assume policy is described by the following instrument rule:

$$i_t = r_t^n + \rho_i i_{t-1} + (1 - \rho_i)(\phi_\pi \pi_t + \phi_x x_t + v_t).$$

The new parameter ρ_i captures inertia in the central bank's policy rule. When $\rho_i = 0$ we have the basic model analyzed previously. Solve the model for $\rho_i = 0$, 0.25, 0.5, and 0.75. For each case, record the value of the loss function. (If you modify the code used for part (a), remember to remove the planner_objective statement that the discretionary_policy and ramsey_policy commands require.)

(c) How does loss vary with ρ_i? Does inertia in policy improve over discretion? If so, explain why? (Hint: it might be useful set ρ_r and ρ_u equal to zero and compare the impulse responses for different ρ_i to those obtained under optimal discretion and optimal commitment.)

3. Consider the following model for the output gap and inflation at the zero lower bound:

$$x_t = \mathrm{E}_t x_{t+1} + \sigma^{-1}(\mathrm{E}_t \pi_{t+1} + \bar{r} + r_t)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + u_t,$$

where u_t is a mean-zero exogenous stochastic disturbance and r_t is an exogenous stochastic disturbances with mean ρ (ρ is the steady-state real interest rate). Assume initially $r_t = r^{zlb} < 0$ such that $i_t = 0$. r_t follows a Markov process such that with probability q, $r_{t+1} = r^{zlb}$ and with probability $1 - q$, $r_{t+1} = \bar{r} > 0$. Once $r_{t+i} = \bar{r}$, it remains at $\bar{r}$ and i_{t+i} is set to ensure $x_{t+i} = \pi_{t+i} = 0$.

(a) Assume that $u_t \equiv 0$ for all t. If $r_t = r^{zlb}$, solve for equilibrium values of x_t and π_t as functions of q.

(b) How does an increase in q affect x_t and π_t. Explain.

(c) If π_t is interpreted as inflation relative to target (2% in the U.S.), one could argue that from 2009 to 2016 the U.S. was characterized by $i_t = 0$, $x_t < 0$, and $\pi_t = 0$. Is such an outcome consistent with your results in part (a)? If $u_t \neq 0$, are there values of the cost shock that would be consistent with $x_t < 0$ and $\pi_t = 0$? Does it require a positive or negative cost shock? Explain.

4. Compare two economies that are identical to the economy in the previous question except that one has more flexible prices, so κ is larger for the economy with more flexible prices. How do the output gaps and inflation rates you found in part (a) compare in these two economies. Which one has the lower output gap at the zero lower bound? Explain.

5. Forward guidance (Based on Carlstrom, Furest, and Paustrian 2012). Suppose the economy is represented by a simple NK model consisting of a standard consumption Euler condition and NK Phillips curve:

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

where r_t is an exogenous stochastic disturbance (the only disturbance in the model). Assume from t until $t+N$, $r_{t+i} < 0$ such that the economy is at the ZLB. From $t+N+1$ on, $r_t = \bar{r} > 0$. The central bank credibly commits to keeping the nominal interest rate at zero until $t+N+M$ where $M \geq 0$. For $t+N+M+k$, $k \geq 1$, the monetary authority follows a targeting rule of the form $\pi_t + \psi x_t = 0$. For calibration purposes, set $\sigma = 1$, $\beta = 0.99$, and $\kappa = 0.08$. Set $N = 3$. From t until $t+N$, $r = -0.02$, while $\bar{r} = 0.03$.

(a) Using Matlab (or Excel for that matter), plot the equilibrium for x_{t+i} and π_{t+i}, $i = 0, 1, ..., N+M+3$ for $M = 0, 1, 2$.

(b) Is forward guidence effective in boosting output at the zero lower bound? Briefly explain what is going on.