Economics 205C
Spring 2017

Problem Set 1: Due in class, Tuesday, April 18

Group work is fine, but if you work with a group, put the names of all members of the group on the assignment you turn in.

1. The Euler condition for optimal consumption in a basic NK model takes the form

$$C_t^{-\sigma} = \beta \mathrm{E}_t \left(\frac{1+i_t}{1+\pi_{t+1}} \right) C_{t+1}^{-\sigma}, \tag{1}$$

where the utility of consumption is $C_t^{1-\sigma}/(1-\sigma)$.

   (a) Linearize this condition around a zero-inflation steady state.

   (b) Now assume households display habit persistence, with the utility of consumption given by $(C_t - hC_{t-1})^{1-\sigma}/(1-\sigma)$, $h > 0$. Derive the Euler condition for the optimal intertemporal allocation of consumption in the following two cases:

      i. External habit persistence – the household takes C_{t-1} as given by aggregate consumption when it chooses C_t.

      ii. Internal habit persistence – the household treats C_{t-1} as its own past consumption when it chooses C_t.

   (c) For the case of part b.i, derived the linearized Euler condition. How does it compare to (1)? Explain.

2. Consider the following basic NK model:

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1} \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t \right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + u_t$$

$$u_t = \rho u_{t-1} + v_t,$$

where x is the output gap, i is the nominal interest rate, π is the inflation rate, r is the equilibrium real interest rate when prices are flexible, and u is a cost shock. Assume r and v are exogenous stochastic iid disturbances. Assume monetary policy is given by

$$i_t = \phi \pi_t.$$

   (a) Write the model in the form

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = M \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + N \begin{bmatrix} r_t \\ u_t \end{bmatrix}.$$

   Give explicit expressions for the 2×2 matrices M and N.

(b) The Blanchard-Kahn conditions for a locally unique rational expectations solution require that both eigenvalues of M lie outside the unit circle. Assume $\sigma = 1$, $\eta = 1$, $\beta = 0.99$, $\rho = 0.9$, and $\kappa = (1-\omega)(1-\beta\omega)(\sigma+\eta)/\omega$, where ω is the Calvo parameter. Set $\omega = 0.75$. Plot the absolute values of the two eigenvalues of M as functions of ϕ. Verify that the Taylor principle $\phi > 1$ is necessary if the Blanchard-Kahn condition is to be satisfied.

(c) Set $\phi = 1.5$. Solve the model numerically using dynare and plot the responses of the output gap and inflation to a positive realization of v_t.

(d) Repeat part (c) with $\phi = 1.1$. Explain why the responses differ from those you obtained in part (c).

(e) Plot the responses of the output gap and inflation to a positive realization of the cost shock u_t. Do this first for $\phi = 1.5$ and then for $\phi = 1.1$. How do the responses differ as for the two values of ϕ. Explain why thy differ.

3. Repeat part (c) of question 3 using the linearized Euler condition you obtained in part 2.c above if $h = 0.8$. Explain how the responses differ from those obtained in 3.c.

4. Suppose the central bank cares about inflation variability, output gap, variablility and interest rate variability. The objective of the central bank is to minimize

$$\left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda_x x_{t+i}^2 + \lambda_i \left(i_{t+i} - i^*\right)^2\right].$$

The structure of the economy is given by

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t \tag{2}$$

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t\right), \tag{3}$$

where e and r are exogenous stochastic shocks. Let ψ_t denote the Lagrangian multiplier on constraint (2) and let θ_t be the multipler on (3).

(a) Derive the first order conditions for the optimal policy of the central bank under discretion.

(b) Show that θ is non-zero if $\lambda_i > 0$. Explain the economics behind this result.

(c) Derive the first order conditions for the fully optimal optimal commitment policy. How do these differ from the conditions you found in (a)?

(d) Derive the first order conditions for the optimal optimal commitment policy from a timeless perspective. How do these differ from the conditions you found in (c)?

5. In no more than three sentences, explain why the optimal commitment policy in a basic NK model introduces inertia by responding to the lagged value of the output gap.