Economics 205C
Spring 2017

Midterm: Answers

1. Suppose the economy's inflation rate is described by the following equation (all variables expressed as percentage deviations around a zero-inflation steady state):

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{1}$$

where π_t is inflation, x_t is the gap between output and the flexible price equilibrium output level and e_t is a white noise cost shock. The first order condition for the representative household's consumption choice takes the form

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n\right), \tag{2}$$

where r^n is an exogenous white noise process. The central bank sets the nominal interest rate i_t to minimize

$$\frac{1}{2}\mathrm{E}_t\left[\sum_{i=0}^{\infty}\beta^i\left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right)\right]. \tag{3}$$

(a) If the loss function (3) is interpreted as a second order approximation to the welfare of the representative household, *explain* what factors determine the optimal weight to put on stabilizing the output gap relative to stabilizing inflation (i.e., how does λ depend on structural characteristics of the model)? *The key parameters are κ, the output gap elasticity of inflation and θ the price elasticity of demand faced by individual firms. With a higher degree of nominal price rigidity (which implies a smaller κ), a given volatility of inflation generates a larger degree of price dispersion under the Calvo model as fewer firms are able to adjust each period. This increases the welfare costs of inflation volatility and so reduces λ. However, what also matters is how much households react to relative price dispersion. If θ is large, they respond a lot so the welfare distortion associated with inflation is larger and therefore λ is smaller.*

(b) Ignoring the zero lower bound on nominal interest rates, derive the equilibrium conditions satisfied by x_t and π_t under optimal discretionary policy and show that these conditions are satisfied by $\pi_t = Be_t$ and $x_t = -(\kappa/\lambda)Be_t$ for some constant B. Find the value of B. (Hint: recall that e_t is a white noise process.) *The first order condition under optimal discretion is $\kappa\pi_t + \lambda x_t = 0$. This can be solved with (1) for π_t and x_t. Since e_t is white noise, expected future inflation and the output gap will both equal zero under optimal discretion. Hence, we can solve the following two equations for inflation and the output gap:*

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t = \kappa x_t + e_t$$

$$\kappa \pi_t + \lambda x_t = 0.$$

From the second of these

$$x_t = -\left(\frac{\kappa}{\lambda}\right)\pi_t.$$

Hence,

$$\pi_t = -\left(\frac{\kappa^2}{\lambda}\right)\pi_t + e_t \Rightarrow \pi_t = \left(\frac{\lambda}{\lambda+\kappa^2}\right)e_t,$$

implying $B = / \left(\lambda + \kappa^2\right)$.

(c) In the equilibrium you found in part (b), show that the equilibrium behavior of the nominal interest rate is given by

$$i_t = r_t^n + be_t, \tag{4}$$

where b is a function of the model's parameters. *From (2),*

$$\begin{aligned} i_t &= \mathrm{E}_t\pi_{t+1} + r_t^n + \sigma\left(\mathrm{E}_t x_{t+1} - x_t\right) \\ &= r_t^n - \sigma x_t = r_t^n + \left(\frac{\sigma\kappa}{\lambda}\right)Be_t. \end{aligned}$$

(d) Suppose the central bank considers adopting (4) as its policy rule for setting i_t. What problems might arise if such a policy rule is adopted? How can these problems be avoided? Explain. *The policy rule given by (4) does not satisfy the Taylor Principle as i_t is not adjusted to endogenous variables. So it does not ensure there is a locally unique, stationary rational expectations equilibrium. For example, if for some reason expected inflation rose, the nominal interest rate is not adjusted, so the real interest rate falls, causing the output gap to rise leading inflation to rise and allowing for self-fulfilling rational expectations equilibria.*

2. Assume the economy can be characterized by the following three equation system:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t\pi_{t+1} - r_t^n\right)$$

$$\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \kappa x_t$$

$$i_t = \max\left\{\begin{array}{c} 0 \\ r_t^n + \phi\pi_t;\ \phi > 1 \end{array}\right. .$$

In periods $t = 1$ and 2, $r_1^n = r_2^n = r^{zlb} < 0$ and r^{zlb} is sufficiently negative such that $i_1 = i_2 = 0$. At $t = 3, ...,$ $r_t^n = r > 0$ and $i_t = r_t^n + \phi\pi_t$.

(a) Verify that the equilibrium inflation rate and output gap for $t = 3, 4, ...$ equal zero. *At $t \geq 0$, equilibrium is defined by*

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(\phi\pi_t - \mathrm{E}_t\pi_{t+1}\right)$$

$$\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \kappa x_t.$$

These two equations are satisfied by $\pi = x = 0$ for all $t \geq 3$. So this is an equilibrium. Since the Taylor Principle is satisfied, it is the locally unique equilibrium.

(b) Given that $\pi_t = x_t = 0$ for $t \geq 3$, what are the equilibrium values of x_2 and π_2? *At $t = 2$, $i_2 = 0$ and $r = r^{zlb}$. Therefore, π_2 and x_2 satisfy*

$$x_2 = \mathrm{E}_2 x_3 + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_2\pi_3 + r^{zlb}\right) = \left(\frac{1}{\sigma}\right)r^{zlb} < 0$$

and

$$\pi_2 = \beta\mathrm{E}_2\pi_3 + \kappa x_2 = \kappa x_2 = \left(\frac{\kappa}{\sigma}\right)r^{zlb} < 0.$$

(c) Given your answer to part (b), what are the equilibrium values of x_1 and π_1?*At $t = 1$, $i_1 = 0$ and $r = r^{zlb}$. Therefore, π_1 and x_{11} satisfy*

$$\begin{aligned} x_1 &= \mathrm{E}_1 x_2 + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_1 \pi_2 + r^{zlb}\right) = \left(\frac{1}{\sigma}\right) r^{zlb} + \left(\frac{1}{\sigma}\right)\left[\left(\frac{\kappa}{\sigma}\right) + 1\right] r^{zlb} \\ &= \left(\frac{1}{\sigma}\right)\left(2 + \frac{\kappa}{\sigma}\right) r^{zlb}, \end{aligned}$$

and

$$\begin{aligned} \pi_1 &= \beta \mathrm{E}_1 \pi_2 + \kappa x_1 = \beta\left(\frac{\kappa}{\sigma}\right) r^{zlb} + \kappa x_1 \\ &= \beta\left(\frac{\kappa}{\sigma}\right) r^{zlb} + \kappa\left(\frac{1}{\sigma}\right)\left(2 + \frac{\kappa}{\sigma}\right) r^{zlb} \\ &= \left(\frac{\kappa}{\sigma}\right)\left[2 + \beta + \left(\frac{\kappa}{\sigma}\right)\right] r^{zlb}. \end{aligned}$$

(d) Explain how (and why) the degree of nominal price rigidity affects the equilibrium at $t = 1$. *A greater degree of nominal price rigidity reduces the value of κ. This does not affect x_2 – x_2 does depend on expected period 3 inflation, but this is zero, independent of κ. A fall in κ does affect π_2 and reduces the negative impact of r^{zlb} – with the fall in x_2 unchanged, the smaller κ implies this negative output gap affects inflation less, so π_2 falls less than when κ is large. The output gap in the first period is less negative with a smaller κ because with π_2 not as negative, the real interest rate in period 1, equal to $-\pi_2$, is not as high so consumption in period 1 is less negatively affected. With both x_1 and π_2 not as negative, π_1 is not as negative either since it equals $\beta\pi_2 + \kappa x_1$. Note there are three effects on π_1: π_2 is less negative; x_1 is less negative, and κ is smaller so the impact of the output gap on inflation is smaller.*

3. The NK two-country model can be approximated around a zero steady-state inflation rate to obtain

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma_0}\right)\left(i_t - \mathrm{E}_t \pi_{h,t+1} - \tilde{\rho}_t\right), \tag{5}$$

where $x_t = y_t - y_t^f$ is the output gap, $\sigma_0 = \sigma\left[1 + \gamma(1-\sigma)\right]$, and

$$\tilde{\rho}_t \equiv \sigma_0\left(\mathrm{E}_t y_{t+1}^f - y_t^f\right) - \gamma(1-\sigma)\left(\mathrm{E}_t y_{t+1}^* - y_t^*\right).$$

In the definition of $\tilde{\rho}_t$, y_t^* is foreign income. If a_t is a productivity shock, the domestic flex-price output is defined as

$$y_t^f = \frac{\gamma(1-\sigma)\, y_t^* + (1+\eta)\, a_t}{\eta + \sigma + \gamma(1-\sigma)}. \tag{6}$$

Domestic product price inflation is given by

$$\pi_{h,t} = \beta \mathrm{E}_t \pi_{h,t+1} + \bar{\kappa} x_t + e_t, \tag{7}$$

where $\bar{\kappa} = \kappa\left[\eta + \sigma + \gamma(1-\sigma)\right]$ and u_t is an inflation shock. Assume social welfare is given by

$$\frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{h,t+i}^2 + \lambda x_{t+i}^2\right). \tag{8}$$

(a) Carefully explain why is y_t^f independent of foreign income when $\sigma = 1$? *A rise in foreign income has two effects. With more foreign output, its relative price falls (the terms of trade fall). From the perspective of domestic households, there real wage has risen as the CPI (a basket of home produce and foreign produced goods) has fallen. The rise in the real wage induces a substitution effect that leads to an increase in labor supply. This would increase equilibrium employment and domestic output. However, there is also a wealth effect as domestic consumption rises when foreign income increases as foreign and domestic households basically pool income to share consumption risk. This wealth effect acts to reduce labor supply, leading to a fall in equilibrium employment and domestic output. If σ is less than 1, the substitution effect dominates; if $\sigma > 1$, the wealth effect dominates. If $\sigma = 1$, they cancel out.*

(b) If the policy maker wishes to minimize (8), what is the first order condition (after eliminating any Lagrangian multipliers) that characterizes optimal policy under discretion? *The problem is of exactly the same form as in the closed economy, so the first-order condition will be*

$$\kappa\pi_{h,t} + \lambda x_t = 0. \tag{9}$$

(c) Suppose $\sigma = 1$ and suppose y_t^* increases. Under the policy you derived in part (c), what is the effect on the home country's inflation rate as measured by the Consumer Price Index? Explain. *With $\sigma = 1$, y_t^f and $\tilde{\rho}_t$ are independent of σ. So under optimal discretion, (7) and (9) can be jointly solved for $\pi_{h,t}$ and x_t without any knowledge of y_t^*. However, the terms of trade are affected; the rise in y_t^* will reduce the terms of trade of as the price of foreign goods falls. Thus, the consumer price index, given by $p_t = p_{h,t} + \gamma s_t$, falls as $p_{h,t}$ is unchanged and s_t falls..*