Economics 205C
Spring 2017

Midterm: Answer any two (2) questions

1. Suppose the economy's inflation rate is described by the following equation (all variables expressed as percentage deviations around a zero-inflation steady state):

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{1}$$

where π_t is inflation, x_t is the gap between output and the flexible price equilibrium output level and e_t is a white noise cost shock. The first order condition for the representative household's consumption choice takes the form

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n\right), \tag{2}$$

where r^n is an exogenous white noise process. The central bank sets the nominal interest rate i_t to minimize

$$\frac{1}{2}\mathrm{E}_t\left[\sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right)\right]. \tag{3}$$

   (a) If the loss function (3) is interpreted as a second order approximation to the welfare of the representative household, *explain* what factors determine the optimal weight to put on stabilizing the output gap relative to stabilizing inflation (i.e., how does λ depend on structural characteristics of the model)?

   (b) Ignoring the zero lower bound on nominal interest rates, derive the equilibrium conditions satisfied by x_t and π_t under optimal discretionary policy and show that these conditions are satisfied by $\pi_t = Be_t$ and $x_t = -(\kappa/\lambda)Be_t$ for some constant B. Find the value of B. (Hint: recall that e_t is a white noise process.)

   (c) In the equilibrium you found in part (b), show that the equilibrium behavior of the nominal interest rate is given by

$$i_t = r_t^n + be_t, \tag{4}$$

   where b is a function of the model's parameters.

   (d) Suppose the central bank considers adopting (4) as its policy rule for setting i_t. What problems might arise if such a policy rule is adopted? How can these problems be avoided? *Explain.*

2. Assume the economy can be characterized by the following three equation system:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t$$

$$i_t = \max\left\{\begin{array}{c} 0 \\ r_t^n + \phi \pi_t;\ \phi > 1 \end{array}\right. .$$

In periods $t = 1$ and 2, $r_1^n = r_2^n = r^{zlb} < 0$ and r^{zlb} is sufficiently negative such that $i_1 = i_2 = 0$. At $t = 3, ..., r_t^n = r > 0$ and $i_t = r_t^n + \phi \pi_t$.

(a) Verify that the equilibrium inflation rate and output gap for $t = 3, 4, \ldots$. equal zero.

(b) Given that $\pi_t = x_t = 0$ for $t \geq 3$, what are the equilibrium values of x_2 and π_2?

(c) Given your answer to part (b), what are the equilibrium values of x_1 and π_1?

(d) Explain how (and why) the degree of nominal price rigidity affects the equilibrium at $t = 1$.

3. The NK two-country model can be approximated around a zero steady-state inflation rate to obtain

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma_0}\right)\left(i_t - \mathrm{E}_t \pi_{h,t+1} - \tilde{\rho}_t\right), \tag{5}$$

where $x_t = y_t - y_t^f$ is the output gap, $\sigma_0 = \sigma\left[1 + \gamma\left(1 - \sigma\right)\right]$, and

$$\tilde{\rho}_t \equiv \sigma_0\left(\mathrm{E}_t y_{t+1}^f - y_t^f\right) - \gamma\left(1 - \sigma\right)\left(\mathrm{E}_t y_{t+1}^* - y_t^*\right).$$

In the definition of $\tilde{\rho}_t$, y_t^* is foreign income. If a_t is a productivity shock, the domestic flex-price output is defined as

$$y_t^f = \frac{\gamma\left(1 - \sigma\right) y_t^* + \left(1 + \eta\right) a_t}{\eta + \sigma + \gamma\left(1 - \sigma\right)}. \tag{6}$$

Domestic product price inflation is given by

$$\pi_{h,t} = \beta \mathrm{E}_t \pi_{h,t+1} + \bar{\kappa} x_t + e_t, \tag{7}$$

where $\bar{\kappa} = \kappa\left[\eta + \sigma + \gamma\left(1 - \sigma\right)\right]$ and u_t is an inflation shock. Assume social welfare is given by

$$\frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{h,t+i}^2 + \lambda x_{t+i}^2\right). \tag{8}$$

(a) Carefully explain why is y_t^f independent of foreign income when $\sigma = 1$?

(b) If the policy maker wishes to minimize (8), what is the first order condition (after eliminating any Lagrangian multipliers) that characterizes optimal policy under discretion?

(c) Suppose $\sigma = 1$ and suppose y_t^* increases. Under the policy you derived in part (c), what is the effect on the home country's inflation rate as measured by the Consumer Price Index? Explain.