Economics 204C **Midterm 3** **Spring 2017**

1. Suppose that the cost of building a bridge is \$94K and if the bridge is built, Player1 will have a tax payment (not transfer payment) of \$62K, Player 2 will have a tax payment of \$32K and Player 3 will pay nothing. Suppose that the true value to Players 1, 2 and 3 is \$40K, \$35K and \$55K. Under the VCG mechanism,

A. Will the bridge be built? (1)

B. What will be the transfer payments? Show your calculations. (6)

C. Why is truth a dominant strategy equilibrium (Explain in words). (4)

2. Gibbard-Satterthwaite:

If $f(\theta)$ has universal domain and the range has at least three elements, then the social choice function is truthfully implementable in dominant strategies if and only if it is dictatorial. Provide an example illustrating both aspects of this proposition along with an accompanying explanation. Note that quasi-linearity is not being assumed. (6)

3. Let θ_i be independently and uniformly distributed on [0,1]. Consider a sealed-bid, all-pay private-values auction.

(A) Derive the bidding functions. This requires solving the differential equations, which is easy to do in this case. (8)

(B) Determine the expected revenue based on your results in (A) (4)

(C) Show that the monotonicity requirement is satisfied. (2) Turn page over

4. The revenue equivalence theorem:

HOTELLING'S LEMMA: Suppose that $\theta_i \in [0,1]$. If $V^*(\theta_i) = V^i(\theta_i, \theta_i) = u^i(f(\theta_i, \theta_{-i}), \theta_i)$ is absolutely continuous, then

(a) $V^*(\theta_i) = V^i(\theta_i, \theta_i) = u^i(f(\theta_i, \theta_{-i}), \theta_i) = u^i(f(0, \theta_{-i}), 0) + \int_0^{\theta_i} u_2^i(f(s, \theta_{-i}), s)\,ds$

(b) If V^* is differentiable at θ_i, then $V^{*\prime}(\theta_i) = V_2^i = u_2^i$

Assume that the following utility function, $u^i(f(m_i, \theta_{-i}), \theta_i) = P^i(m_i, \theta_{-i})\theta_i - t^i(m_i, \theta_i)$, is absolutely continuous.

EXPECTED REVENUE EQUIVALENCE THEOREM. Let there be N risk neutral buyers. Each buyer's valuation, θ_i, is drawn independently from an interval [0, 1] with strictly positive density, $g(\theta_i)$. Suppose that a given pair of Bayesian Nash equilibria of two different auction procedures are such that for every buyer i: (1) for each possible realization of θ_i, buyer i has an identical probability of getting the good in the two auctions; and (2) buyer i has the same expected utility level in the two auctions when his valuation for the object is the lowest possible level. Then the equilibria of the two auctions generate the same expected revenue for the seller.

Making use of Hotelling's lemma, prove the Expected Revenue Equivalence Theorem. (12)