

(1) Each firm has a differentiated product and constant marginal costs. Firm  $i$ 's profit, when it charges price  $p_i$  and its rivals' price vector is  $p_{-i}$ , is  $\Pi^i(p_i, p_{-i}; y) = (p_i - c_i)D^i(p_i, p_{-i}; y)$ , where  $c_i$  is  $i$ 's unit cost and  $D^i$  is  $i$ 's demand function.  $y$  stands for the exogenous variable income in this partial equilibrium model. Assume that firm  $i$ 's elasticity of demand is a strictly-decreasing function of firm  $j$ 's price for all  $i \neq j$ . Using monotone comparative statics, find the effect of a change in  $c_i$  on  $p_i$  and on all  $p_j$  in  $p_{-i}$ . Be very explicit about your results. (16)

(2) When there are strategic substitutes, an equilibrium does not exist in general. However, there are two important exceptions. What are they? (4)

(3) 1. ARROW THEOREM: It is impossible to have a social welfare functional that satisfies all of the following conditions:

1. (T) It is complete, reflexive and transitive
2. (P) It satisfies the Paretian principle.
3. (I) It satisfies independence of irrelevant alternatives
4. (ND) There is no dictator
5. (U) Unrestricted domain.

Making use of the Lemma that if everyone ranks  $Y$  either at the top or the bottom, society must rank  $Y$  at either the top or the bottom, prove the rest of the theorem. (14)

4. A. What is the mathematical expression for a social welfare function when there are 2 individuals in the society? (2)
- B. Define Anonymity (equivalently symmetry) in the context of the above social welfare function. (2)
- C. Provide the formula for the Pareto Principle in the context of the above social welfare function. (2)