ECON 204B-Advanced Microeconomics **UCSC**
Homework 4-Answer Key

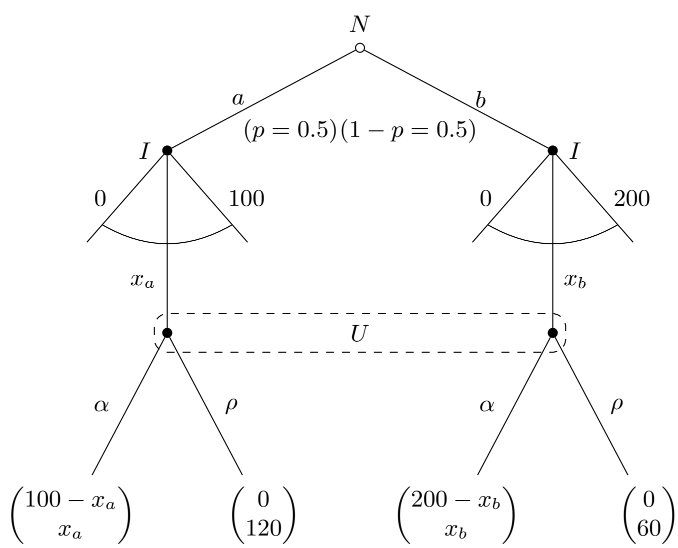
Part I. Problems

1. **Problem 1**

(a) Assume that the game procedure is common knowledge. Of course, neither player will trade when their payoff is higher than other's. Player B knows that player A's maximum payoff is 1000, so he or she will not trade when $y > 1000$. Player A also knows this and he will not trade when there is $x > 500$ in his or her envelope. Player B knows this and he will not trade when $y > 500$. Anticipating B's refusal to trade when $y > 500$, Player A will not trade when $x > 250$. Iterating this decision process, trade occurs with positive probability only when $x = y = 0$.

(b) The student is using prior probabilities, implicitly assuming that trade will occur with probability 1 which, as we have seen, is incorrect. She or he apparently overlooked the logic of iterated dominance and Bayes Theorem.

2. **Problem 2**
Denote α as acceptance and ρ as rejection.



(a) If $x_a = x_b$, $\mu_U(a|x_s) = \mu_U(a) = \frac{1}{2}$ and $\mu_U(b|x_s) = \mu_U(b) = \frac{1}{2}$. Let $x_S \equiv x_a = x_b$. This is case of pooling.

$$\pi_U(\alpha) = \frac{1}{2}(x_S) + \frac{1}{2}(x_S) = x_S$$

$$\pi_U(\rho) = \frac{1}{2}(120) + \frac{1}{2}(60) = 90$$

Thus, player U will accept the bid if $x_S \geqq 90$, i.e. $BR_U(x_S \geqq 90) = \alpha$, and player U will reject the bid if $x_S \leq 90$, i.e. $BR_U(x_S \leq 90) = \rho$.

When player I observes type a, he or she will bid $x_S \leqq 100$. When he or she observes type b, will bid $x_S \leqq 200$.

Therefore, the PBE outcomes in which $x_a = x_b$ are $(90 \leqq x_s \leqq 100, \alpha_{x_s}, \mu(a|x_s) = \frac{1}{2})$ and $(x_s < 90, \rho_{x_s}, \mu_U(a|x_s) = \frac{1}{2})$.

Note: A full specification of the PBE should also include beliefs and actions following off-equilibrium play. This PBE is not very sensitive to such beliefs and actions.

(b) If $x_a \neq x_b$, $\mu_U(a|x_a) = 1$ and $\mu_U(b|x_b) = 1$. This is case of separating PBE.

$$\pi_U(\rho|x_a) = 120 \qquad \textit{and} \qquad \pi_U(\alpha|x_a) = x_a$$
$$\pi_U(\alpha|x_b) = x_b \qquad \textit{and} \qquad \pi_U(\rho|x_b) = 60$$

Thus, player U will reject the bid x_a if $x_a < 120$, and player U will accept the bid x_b if $x_b > 60$. Knowing this, player I has an incentive to deviate in state a if $x_b \in [60, 100)$. So such bids are not part of a BNE.

When player I observes type a, he or she will bid $x_a \leqq 100$. When he or she observes type b, will bid $x_b \in [100, 200]$.

Therefore, the PBE outcomes in which $x_a \neq x_b$ and x_a is rejected and x_b is accepted are $((x_a \leqq 100 \leqq x_b \leqq 200), (\rho_{x_a}, \alpha_{x_b}), \mu_U(a|x_a) = 1)$.

Again, the full specification should include posterior beliefs off the equilibrium path, but these are arbitrary within a wide range, as in the answer to the next part.

Since player I does not bid x_a over 100, there is no PBE in which $x_a \neq x_b$ and x_a is accepted and x_b is rejected.

(c) When state a realizes, $x_a > 100$ yields negative profit for I if the offer is accepted. So this strategy is weakly dominated by $x_a < 100$. On the other hand, $x_a > 100$ is not weakly dominated when state is b. Thus, U thinks that if $x > 100$, the state is b for sure.

In this case, I has an incentive to deviate if $x_b > 100$. That is because, as long as $x > 100$, the offer will be accepted, so I will offer lower prices than x_b and gain higher payoff. Thus, in an equilibrium, $x_b = 100$.

Also, I has an in centive to deviate at state b if lower prices will be accepted, so in an equilibrium, the best responses for $x' < x_b$ must be reject.

Separating equilibria in this case are

$$x_a < 100,\ x_b = 100,\ a(x_a) = R,\ a(x_b) = A,\ a(x') = \begin{cases} R & (x' < 100) \\ A & (x' > 100) \end{cases},$$
$$\mu(a|x_a) = 1,\ \mu(a|x_b) = 0,\ \mu(a|x') = \begin{cases} q > \frac{x'-60}{60} & (x' < 100) \\ q = 1 & (x' > 100) \end{cases} \tag{17}$$

3. **Problem 3**

(a)

$$WTP_L = -[-1 - \frac{1}{2} \cdot 0.2 \cdot 1] = 1.1$$
$$WTP_H = -[-2 - \frac{1}{2} \cdot 0.2 \cdot 16] = 3.6$$

(b)

$$P^* = (0.8 \cdot 1 + 0.2 \cdot 2) = 1.2$$

(c) If SI charges 0.4 above the break-even point, $P_A^* = 1.6$. Only high risk students would find it worthwhile to join SI.

(d) SI's profit in case c is

$$\Pi_A = (1.6 - 2) \cdot (20000 \cdot 0.2)$$
$$= -1600.$$

To increase there profit, they could discriminate the price for each type of students. If students' types are observable, they would set the premium as 1.1 to low risk students and 3.6 to high risk students. Then, SI's profit is

$$\Pi_{PD} = (3.6 - 2) \cdot (20000 \cdot 0.2) + (1.1 - 1) \cdot (20000 \cdot 0.8)$$
$$= 8000.$$

Of course, the types are not normally observable. In this case, SI could offer two plans: Plan A with a high premium and low deductible that would attract the high risk students, and Plan B with a lower premium and higher deductible that would attract the low risk students. Profits would be at most 8000.

4. **Problem 4**

(a) Professor P's utility maximization probelm is as the following.

$$\max_{x,s} x - s$$

subject to

$$s - \frac{1}{2}x^2 \geqq 0.$$

Substitute the constraint into professor's payoff function and differentiate it with respect to x.

$$1 - x^* = 0$$
$$\therefore x^* = 1 \text{ and } s^* = 0.5$$

(b) Let's first take a look at Mr. A's choice of x.

$$\max_{x \geqq 0}(ax+b) - \frac{1}{2}x^2$$

The first order condition shows $x^* = a$. $a \geqq 0$ and $b \geqq -\frac{1}{2}a^2$ has to be satisfied. Now, P will choose a and b that maximize her utility.

$$\max_{a \geqq 0, b \geqq -\frac{1}{2}a^2} x - (ax+b) = a - a^2 - b$$

By the first order conditions, $a^* = 1$ and $b^* = -\frac{1}{2}$. Therefore, $x^* = 1$ and $s^* = 0.5$.

(c) P could not obtain higher utility using a non-linear wage schedule, because the solution using the linear wage schedule is the highest utility she could obtain.

5. **Problem 5**
The child maximizes his income, $V = I_C(A) + B$, choosing A and taking into account that his action affects the bequest. The first order condition is

$$\frac{dI_C(A)}{dA} + \frac{dB}{dA} = 0.$$

The parent maximizes $W(U,V) = W(I_P(A) - B, I_C(A) + B)$ choosing B. The first order condition is

$$-W_U(I_P(A) - B, I_C(A) + B) + W_V(I_P(A) - B, I_C(A) + B) = 0.$$

We can think B as a function of A. Differentiate the first order condition with respect to A.

$$-W_{UU} \cdot (\frac{dI_P(A)}{dA} - \frac{dB}{dA}) - W_{UV} \cdot (\frac{dI_C(A)}{dA} + \frac{dB}{dA})$$
$$+W_{VU} \cdot (\frac{dI_P(A)}{dA} - \frac{dB}{dA}) + W_{VV} \cdot (\frac{dI_C(A)}{dA} + \frac{dB}{dA}) = 0.$$
$$(W_{UU} - W_{VU}) \cdot (\frac{dI_P(A)}{dA} - \frac{dB}{dA}) = (W_{VV} - W_{UV}) \cdot (\frac{dI_C(A)}{dA} + \frac{dB}{dA})$$

By the child's first order condition, the equation above becomes

$$(W_{UU} - W_{VU}) \cdot (\frac{dI_P(A)}{dA} - \frac{dB}{dA}) = 0.$$
$$\therefore \frac{dI_P(A)}{dA} = \frac{dB}{dA}$$

Substitute this into the child's first order condition.

$$\frac{dI_C(A)}{dA} + \frac{dI_P(A)}{dA} = 0.$$

This implies that the child chooses the A that maximizes the family aggregate income, $I_C(A) + I_P(A)$.

The first-best outcome from the parent's perspective is to maximizes her objective function. The first order conditions for the first-best outcomes are, with respect to B and A respectively,

$$\begin{aligned} W_U &= W_V, \\ W_U \cdot \frac{dI_P(A)}{dA} + W_V \cdot \frac{dI_C(A)}{dA} &= 0. \\ \therefore \frac{dI_P(A)}{dA} + \frac{dI_C(A)}{dA} &= 0. \end{aligned}$$

Thus, the first-best outcome from the parent's perspective is to maximize the family's aggregate income.

Part II. Textbook problems

6. **13.B.3**

(a) Suppose firms offer a wage of w. All workers of type θ, with $r(\theta) \leqq w$, will accept the wage and work. Suppose there exists a θ^* with $r(\theta^*) = w$. Then all workers of type $\theta \geqq \theta^*$ will work, since $r(\theta) \leqq r(\theta^*) = w$ and r is decreasing. Thus, the more capable workers are the ones who will work at any given wage.

(b) Firms can offer the wage $w = \bar{\theta}$, and since $r(\bar{\theta}) > \bar{\theta}$ no workers of type $\bar{\theta}$ will work. No worker of any type will work. Therefore, the competitive equilibrium is Pareto efficient.

(c) If $w = \hat{\theta}$, only workers of type $\theta \geqq \hat{\theta}$ will accept the wage w and work. But $E[\theta|\theta \geqq \bar{\theta}] > \hat{\theta} = w$, which implies that firms demand more workers than there are in supply, and the market will not clear. If $w < \hat{\theta}$, only workers of type $\theta \geqq \theta^* > \hat{\theta}$, with $r(\theta^*) = w$, will accept the wage w and work.

Thus, to obtain market clearing, firms have to offer a wage $w > \hat{\theta}$, which implies that some workers of type $\theta < \hat{\theta}$ will accept the job, and there is over employment in the competitive equilibrium.

7. **13.C.4**
The workers' utility maximization problem is the following.

$$\max_e w(e) - c(e,\theta)$$

where

$$c(e,\theta) = \frac{e^2}{\theta}.$$

The first order condition is $w'(e) = \frac{2e^*}{\theta}$. Since the firms are competitive, $w(e^*) = \theta^*$ in equilibrium. Therefore, $w'(e) = \frac{2e^*}{w(e^*)}$. Thus, $w(e) = \sqrt{2}e$. So the unique separating PBE is $(e(\theta) = \frac{\theta}{\sqrt{2}}, w(e) = \sqrt{2}e, \mu(\theta|e) = \sqrt{2}e)$.

8. **14.B.3**

(a)

$$\begin{aligned} Eu &= E[\alpha + \beta\pi|e] - \phi Var[\alpha + \beta\pi|e] - g(e) \\ &= \alpha + \beta E[\pi|e] - \phi\beta^2 Var[\pi|e] - g(e) \\ &= \alpha + \beta e - \phi\beta^2\sigma^2 - g(e) \end{aligned}$$

(b) Optimal risk sharing will result in a fixed wage for the agent, and maximizing the principal's profits ensures that this wage will exactly compensate the agent for his effort. Therefore, the first-best contract with observable effort is the solution to the following.

$$\max_e E[\pi|e] - g(e)$$

By the first order condition, $g'(e^*) = 1$, which gives us the optimal effort level e^*, and $w = g(E^*)$ is the wage.

(c) The principal's problem can be divided into two steps. First, for a given effort level e', the optimal individually rational and incentive compatible compensation scheme is as the following.

$$\min_{\alpha,\beta} \alpha + \beta e'$$

subject to

$$\begin{aligned} &\alpha + \beta e' - \phi\beta^2\sigma^2 - g(e') \geqq 0 \\ &\alpha + \beta e' - \phi\beta^2\sigma^2 - g(e') \geqq \alpha + \beta e - \phi\beta^2\sigma^2 - g(e) \forall e \neq e' \end{aligned}$$

The second constraint implies that given e', $\beta e - g(e)$ should reach a maximum at e'. The condition of the question allow us to replace the second constraint with the first order condition $g'(e') = \beta$, which uniquely deteremines β given e'. Thus, $\alpha =$

$g(e') + \phi g'(e')^2\sigma^2 - g'(e')e'$. We can now find the optimal compensation scheme by solving:

$$\max_e E[\pi|e] - E[g(e) + \phi g'(e)^2\sigma^2 - g'(e)e + g'(e)\pi|e]$$
$$= \max_e e - g(e) - \phi g'(e)^2\sigma^2$$

Given the conditions of the question, this is a concave program, so the first order condition yields $g'(e) = \frac{1}{1+2\phi\sigma^2 g''(e)}$. This implies that $0 < \beta < 1$, which is reasonable because the incentives are not fully aligned with profits due to optimal risk sharing. As ϕ increases, the agent is more averse to risk and therefore β will be lower. This is same as σ^2 increases.

9. **14.C.8**

(a) The indifference curves of the two types and the firm's isoprofit curve are depicted in figure 1.

The problem that Shangri La would want to solve is:

$$\max_{P_T, W_T, P_B, W_B} \Lambda P_T + (1-\Lambda)P_B$$

s.t.

$$(i)\theta_T P_T + W_T \leqq v$$
$$(ii)\theta_B P_B + W_B \leqq v$$
$$(iii)\theta_T P_T + W_T \leqq \theta_T P_B + W_B$$
$$(iv)\theta_B P_B + W_B \leqq \theta_B P_T + W_T$$
$$(v)P_T, W_T, P_B, W_B \geqq 0$$

(b) In the program above, constraint (i) and (iv) together with $\theta_B < \theta_T$, imply that constraint (ii) is redundant, so it is never binding. Constraint (i) therefore must bind for if it would not, we can reduce P_T and P_B by $\epsilon > 0$, and all the remaining constrainits will still be satisfied. This implies that tourists will be indifferent between buying and not buying a ticket.

(c) Assume that $(P_T, W_T), (P_B, W_B)$ is an optimal, incentive and assume in negation that $W_B > 0$. Now reduce $W_B$by $\epsilon > 0$, and increase P_B by $\frac{\epsilon}{\theta_B}$ so that B type's utility does not change, and the firm earns higher profits from the B type. We need to check that the T Type will not choose this new compensation package. Indeed,

$$\theta_T P_T + W_T \leqq \theta_T P_B + W_B = \theta_T(P_B + \frac{\epsilon}{\theta_T}) + (W_B - \epsilon) < \theta_T(P_B + \frac{\epsilon}{\theta_B}) + (W_B - \epsilon)$$

contradicting that $(P_T, W_T), (P_B, W_B)$ is an optimal, incentive compatible contract. Therefore, we must have $W_B = 0$. If, in an optimal contract, the business travellers

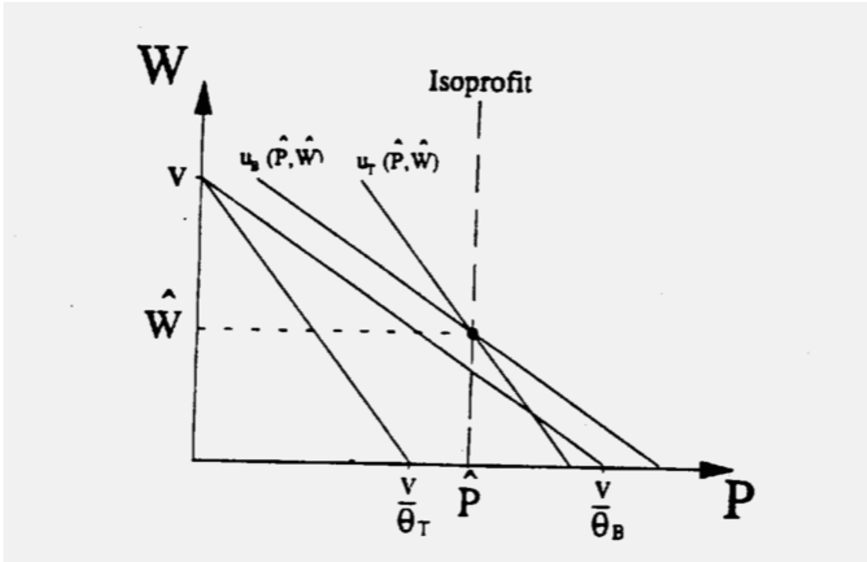


Figure 1: Indifference curves and isoprofit curve

were not indifferent between (P_T, W_T) and (P_B, W_B), we could slightly raise P_B and all the constraints would remain satisfied, and the firm would earn higher profits from the business types. Therefore in an optimal contract we must have the business types indifferent between $(P_T, W_T), (P_B, W_B)$.

(d) By lowering P_T and increasing W_T to keep the tourists indifferent betwen buying a ticket or not buying one, the firm can increase P_B. We know from parts (b) and (c) that if W_T is raised by ε and P_T is lowered by $\frac{\varepsilon}{\theta_T}$ (thus holding the tourists indifferent about buying), then P_B must be increased by $\frac{\varepsilon(\theta_T-\theta_B)}{\theta_T\theta_B}$ to keep the business types indifferent between their own package and the new package. The linearity of this trade off allows it to hold true anywhere in (P,W) space, implying it will be profitable if and only if:

$$\lambda\frac{\varepsilon}{\theta_T} < (1-\lambda)\cdot\frac{\varepsilon(\theta_T-\theta_B)}{\theta_T\theta_B}$$

which can be simplified to

$$\frac{\lambda}{1-\lambda} < \frac{\theta_T-\theta_B}{\theta_B}$$

We can see that this condition is indepent of cost c. Therefore, the price discrimination has two cases:
(1) If $\frac{\lambda}{1-\lambda} < \frac{\theta_T-\theta_B}{\theta_B}$, only the high types will be served and the scheme will be the following (assuming $c < \frac{v}{\theta_B}$):

$$(P_t, W_T), (P_B, W_B) = (0, v), (\frac{v}{\theta_B})$$

(2) If $\frac{\lambda}{1-\lambda} > \frac{\theta_T - \theta_B}{\theta_B}$, all types will be served and we will have a pooling scheme, assuming $c < \frac{v}{\theta_T}$:

$$(P_T, W_T) = (P_B, W_B) = (\frac{v}{\theta_T}, 0)$$

We can see that if the proportion of B types is large enough (or λ small enough) the firm will choose to serve B types strictly. Additionally, if θ_B is sufficiently small, the firm is more likely to serve only B types. If θ_T is sufficiently small, the firm is more likely to serve T types as well as B types.

(e) The firm will serve only B types if the conditions $c < \frac{v}{\theta_B}$ and case (1) from part (d) are met. If we are in case (2) of part (d) and $\frac{v}{\theta_T} < c < \frac{v}{\theta_B}$, then the scheme from part (d) would be sub-optimal as the firm would lose money. As such, the firm would choose the scheme in case (1) and, again, only serve B types. If $c > \frac{v}{\theta_B}$, the firm will simply choose to not operate.