Problem Set #4

Instructions. Due in class Tuesday, March 14. As usual, please acknowledge the major contributors to the solution of each problem, and the time you invested. Feel free to rate the apparent return on that investment.

Part I. Problems. When insufficient information is provided, write down a plausible specific assumption and proceed to the solution.

1. There are two risk neutral players, A and B. Nature chooses an amount of money, x, from a uniform distribution over [0, 1000], and places it in an envelope for A. Nature then tosses a fair coin. If it is heads, Nature places y = 2x in an envelope for B. If it's tails, Nature places y = x/2 in an envelope for B. The players (who know that a coin was tossed but not how it came out) look at their envelopes in private, and then announce "trade" or "no trade" simultaneously. Trade occurs if and only if both announce "trade".
a. Is there an equilibrium in which trade occurs with positive probability? [Hint: to get started, think about whether there are high values of x or y that preclude trade. You might want to use the logic of iterated dominance.]
b. A philosophy student tells you that he has heard of this problem, and insists that trade will occur with probability 1. His reasoning is that either player will look at the amount z in her envelope, compute her post-trade value as 0.5(z/2) + 0.5(2z) = 1.25z > z, and so regard trade as personally beneficial; indeed, ex ante, the trading opportunity is mutually beneficial, which any economist should understand. What, if anything, is wrong with the philosophy student's reasoning?

2. An uninformed player U possesses an object that is either type a (and worth $120 to him) or else type b (and worth $60 to him). U doesn't know the true type and regards them as equally likely. By contrast, the other player I does know the true type but values type a at $100 and type b at $200. Consider a one-shot game in which I bids a chosen amount x, and U either accepts or rejects that bid. Then the true type is revealed to U and the players receive their final payoffs.

In your analysis of this game, assume that x_k (denoting I's bid when the commodity is of type k = a, b) is an integer, and confine your attention to pure strategy Perfect Bayesian Equilibria. Recall that a PBE includes both a strategy profile and beliefs at all information sets, including those never reached in equilibrium. When the same outcome is generated by different PBEs, you need only specify one of the PBEs fully.

a. Find all the PBE outcomes in which $x_a = x_b$. Explain why you think you have found *all* the pooling PBE outcomes.
b. Find all the PBE outcomes in which $x_a \neq x_b$ and x_a is rejected and x_b is accepted. Again, explain why you have found *all* such separating PBE outcomes. Are there any PBE in which x_a is accepted and x_b is rejected?
c. Assume that I does not play weakly dominated strategies and U knows this. This assumption puts a restriction on out-of-equilibrium beliefs: when U sees an offer that is weakly dominated for one type but not for the other, U should believe with probability 1 it's the other type. Which of the separating PBE found in part b satisfy this assumption?

3. Slug Insurance (SI) is planning to sell policies to 20,000 UCSC and Cabrillo students. 80% of the students are low risk with average health costs (measured in thousands of dollars per year) $C_L = 1$ and standard deviation = 1. The other 20% are high risk with average health costs $C_H = 2$ and standard deviation = 4. All students have risk aversion coefficient $r = 0.2$ in utility function $u(X) = EX - ½ r \text{ Var } X$. A risk-neutral for-profit company, SI has negligible overhead costs. It can't identify the risk type of individual students but does know the 20%-80% breakdown.
a. Compute the willingness to pay for health insurance by each type of student, L and H.
b. What annual premium (P*) would allow SI to break even if all students joined?
c. SI charges 0.4 (or $400) above the break-even point. At that premium, which students would find it worthwhile to join SI?
d. What are SI's profits in case c above? How can SI adjust their strategy to increase profits? How much could they make?

4. Professor P is hiring a teaching assistant, Mr A. Professor P's payoff function is x-s, where x is the number of hours A works and s is the amount she pays A. Mr A's payoff function is $s - x^2/2$; he gets payoff 0 if he doesn't work for P.
a. What choices of x and s will maximize P's utility (subject, of course, to the constraint that A is willing to work for her)?
b. Suppose that P offers a wage schedule $s(x) = ax + b$, where A picks x. What choices of a and b will now maximize her utility?
c. Could P obtain higher utility using a non-linear wage schedule?

5. Consider the following two player game. Player 1 (the child) moves first, and takes an action $A \geq 0$ that produces income $I_c(A)$ for himself, and income $I_p(A)$ for the other player (the parent). The parent observes $I_c(A)$ and $I_p(A)$ and then chooses a bequest B to leave to the child. The child's utility is $V = I_c(A) + B$. The parent's utility from her own consumption is $U = I_p(A) - B$, but she also cares about the utility of the child. The parent maximizes $W(U; V)$ which is strictly increasing, strictly concave, and twice continuously differentiable. The bequest B can be either positive or negative. Prove the *Rotten Kid Theorem*: In SPNE, the child chooses the action that maximizes the family's aggregate income, $I_c(A) + I_p(A)$, even though only he has selfish preferences. Is this the first-best outcome from the parent's perspective?

II. Textbook problems.
Write up and turn in your solutions to MCWG problems 13.B.3, 13.C.4, and 14.B.3; for extra credit do 14.C.8.

III. Short essay. Write briefly (about 100 words) for an audience whose technical background is similar to yours.

What is the difference between signaling and screening? What problems do they both solve, and when is signaling more likely to offer a better solution?