Problem Set #3

Instructions. Due in class Tuesday February 28. Don't forget to **give credit** to anyone who helped and state how much of your own time you spent on each problem. You might also comment on the perceived return: how much did you seem to learn per hour?

Part I. Problems. When insufficient information is provided, write down a plausible specific assumption and proceed to the solution.

1. Consider a two player repeated game in which, at each stage, player 1 has two pure strategies and player 2 has three pure strategies.
    a. How many different action profiles can be observed in the first stage?
    b. How many different pure strategies does each player have at stage 2?
    c. How many different action histories can there be after 3 stages of play?
    d. How many different pure strategies does each player have at stage 4?

2. An investor (Player 1) spends an amount $k \in [0,20]$ of her capital on a project in a foreign country. Its government (Player 2) then decides on the tax rate $t \in [0,1]$ for the project. The productivity is $r>0$ so after investment the project is an asset worth $A = (1+r)k$. The government's payoff is $T=tA$ and the investor's payoff is $U = c_0 + c_1$ where $c_0 = 20-k$ and $c_1 = (1-t)A$.
    a. For what values of t will a rational investor choose $k>0$?
    b. What is the government's best-response to $k>0$?
    c. Find the SPNE of this static game, and the corresponding payoff vectors.
    d. Suppose instead that the government commits to t before the investor's decision k. Find the SPNE of this EFG where the government moves first.
    e. Find efficient values t^* and k^*, i.e., values that maximize the payoff sum in either EFG.
    f. Write out the NFG in which the investor chooses either $k=0$ or $k=k^*$, and the government chooses either $t=1$ or $t=t^*$. Find the NE of this game.
    g. Take the game in part f as the stage game in an infinitely repeated game with discount factor $d<1$. Generally speaking, what factors determine the discount factor d? In this particular situation, which specific factors are especially relevant for d?
    h. For which values of d can there be NE grim trigger strategies in the repeated game that sustain an efficient stage game outcome?
    i. Using your answers in previous parts, predict the circumstances under which the outcome of the foreign investment game is efficient or inefficient.

3. Consider the symmetric two player game with two alternative stage game strategies, U and D; the payoff to U is 0 if the opponent also plays U and is 4 if he plays D, and the payoff to D is 1 if the opponent plays U and is 2 if he plays D.
    a. Find all evolutionary equilibria of the game in the one population case.
    b. Find all evolutionary equilibria if row players and column players have separate populations. Draw the phase portrait (in the unit square).
    c. Find the phase portrait for case a. inside the unit square.

   d. Interpret your results in terms of equilibrium selection: under which circumstances is each NE of the original 2x2 game a reasonable prediction of the observed outcome?

4. Two countries, A and B, are negotiating on trade. Relative to status quo, the sum of possible gains for the two countries is 10. The gains can be split in any fashion without affecting the total.
   a. Calculate the Nash Bargaining Solution for this trade game.
   b. A third country C enters the trade negotiations. It can provide mutual gains of 15 with country A, or 5 with country B, or net gains of 5 (in addition to the 10 already available) with A and B jointly. Write out the characteristic function for this three-player game.
   c. Is this game convex (as MWG define in their relevant appendix)?
   d. Calculate the core of the three-player game.
   e. Calculate the Shapley value of the three player game.

5. A steel buyer and steel seller form a bilateral monopoly—neither has an alternative exchange partner. The seller has constant marginal cost 10 while the buyer has marginal benefit $210 - q$ on the q^{th} unit purchased. The two players must first agree on price p. Then the quantity traded is the minimum of what the buyer is willing to buy and what the seller is willing to sell at the given price p.
   a. What are the payoffs as functions of p and q?
   b. Find the Pareto frontier in (p, q) space and in payoff space. Identify the point that maximizes the sum of buyer and seller profit.
   c. Suppose the seller chooses price. What is his optimal choice, and what are the resulting profits for buyer and seller?
   d. Suppose the buyer chooses price. What is her optimal choice, and what are the resulting profits for buyer and seller?
   e. What is the Nash Bargaining solution to this problem? Assume that the threat points are extreme points of the Pareto frontier. Hints: find the maximum of the product of buyer and seller "excess" profits, relative to the threat points. Decide whether utility is transferable.

II. Textbook problems. Skim all the problems in MWG Chapter 12. Write up and turn in solutions to 12.B.1, 12.B.3, 12.B.6, 12.D.4 and 12.E.4. For extra credit, try 12.C.14 and/or 12.C.18.

III. Short essay. Write briefly (about 100 words) for an audience whose technical background is similar to yours.

What is your present impression of the usefulness of evolutionary game theory? If you prefer, answer instead the same question about cooperative game theory.