1 Utility

CRRA Utility $u(x|r) = \frac{x^{1-r}}{1-r}$, $r \in (-\infty, \infty)$
CARA Utility $u(x|a) = 1 - e^{-ax}$, $a > 0$
Certainty Equivalent $u(CE) = \int u(x)\, dF(x)$
Risk Premium $u(\int x\, dF(x) - RP) = \int u(x)\, dF(x)$
Absolute Risk Aversion $A(x) = -\frac{u^{"}(x)}{u^{'}(x)}$
Relative Risk Aversion $R(x) = xA(x) = \frac{-xu^{"}(x)}{u^{'}(x)}$
Mean-Variance Approximation
$u(\overline{x}+h) = u(\overline{x}) + (x-\overline{x})u^{'}(\overline{x}) + \frac{1}{2}(x-\overline{x})^2 u^{"}(\overline{x}) + R^3$
$Eu = u(\overline{x}) - \frac{1}{2}A(\overline{x})\sigma_L^2 + R^3$
First Order Stochastic Dominance
G FOSDs F if $F(x) \geq G(x)\ \forall x$
Second Order Stochastic Dominance
G SOSDs F if $\mu_F = \mu_G$ and $\int_{-\infty}^{x} F(t)dt \geq \int_{-\infty}^{x} G(t)dt\ \forall x$

2 Bayes' Theorem

Basic Definitions

$p(s) = \sum_{z \in Z} p(s,z)$ (prior prob. of state s)
$p(z) = \sum_{s \in S} p(s,z)$ (message prob.)
$p(z|s) = \frac{p(s,z)}{p(s)}$ (likelihood)
$p(s|z) = \frac{p(s,z)}{p(z)}$ (posterior prob.)

Bayes theorem

(i) $p(s|z) = \frac{p(z|s)p(s)}{p(z)}$

(ii) $p(s|z) = \frac{p(z|s)p(s)}{\sum_{t \in S} p(z|t)p(t)}$

(iii) $\frac{p(s|z)}{p(t|z)} = \frac{p(z|s)p(s)}{p(z|t)p(t)}$

(iv) $\ln \frac{p(s|z)}{p(t|z)} = \ln \frac{p(z|s)}{p(z|t)} + \ln \frac{p(s)}{p(t)}$

Value of information
$V_I = \sum_{z \in Z} p(z) \sum_{s \in S} p(s|z)[u_z^*(s) - u_0^*(s)]$

3 Normal Form Games

Cookbook for NFG solutions

(i) Get NFG from story or EFG (should be a complete contingency plan)

(ii) Eliminate strictly dominated strategies (never-best-response are the candidates) and reduce the game. If only one profile remains, it is DS solution

(iii) Iterate step(i) until no more dominated strategies, if only one profile remains, it is IDDS

(iv) Inspect for mutual BR $\longrightarrow$ These are pure NE

(v) Check for mixed NE, $\sigma_i \in B_i(\sigma_{-i})$, each $|subset| \geq 2$ of pure strategies for each player, write down the set of simultaneous equation

Payoff function of mixed strategies (2x2)

$$f_1(\sigma_1, \sigma_{-1}) = \sum_{i=1}^{2} p_i \sum_{j=1}^{2} q_j f_1(s_i, t_j)$$

$$where\, \sigma_1 = p_1 s_1 + (1-p_1)s_2, \sigma_{-1} = q_1 t_1 + (1-q_1)t_2$$

Formula for finding mixed strategies (2x2)

$$f_1(s_1, \sigma_{-1}) = f_1(s_2, \sigma_{-1})$$
$$f_2(t_1, \sigma_{-2}) = f_2(t_2, \sigma_{-2})$$

4 Extensive Form Games

Cookbook for solution of perfect information

(i) Convert each penultimate node ν into a terminal node
If ν is owned by player i, then player i choose the maximum payoff
If ν is owned by nature, then take expectation over payoff vectors

(ii) Iterate step 1 until you reach the initial node

(iii) Reconstruct each players strategy for her choices in step 1-2

(iv) The resulting profile is a subgame perfect nash equilibrium (SPNE)

(v) (For imperfect info)Find the smallest subgames that contain terminal nodes. Find all NE of that subgame. Replace the initial node of that subgame by a NE payoff vector. Iterate to a solution $\longrightarrow$ get one SPNE. Then look on step 1 until all NEs in the minimal subgame have been used

5 BNE, PBE and Seq EQ

(i) Beliefs μ_i at each info set for player i are consistent with common prior and likelihood from s_{-i}^* and own realized type $\bar{\theta}_i$ via Bayes

(ii) At each info set, player i max's $E(u_i|\mu_i)$: $\forall\ s_i' \epsilon S_i$
$E_{\theta_{-i}}[u_i(s_i^*(\bar{\theta}_i), s_{-i}^*(\theta_{-i}), \bar{\theta}_i)|\bar{\theta}_i] \geq E_{\theta_{-i}}[u_i(s_i', s_{-i}^*(\theta_{-i}), \bar{\theta}_i)|\bar{\theta}_i]$

(iii) previous items hold in every subgame

(iv) robust to sufficiently small trembles

(i) and (ii) constitue a Bayesian NE
(i) thru (iii) constitue a Perfect Bayesian NE
(i) thru (iv) constitute a sequential equilibrium