Midterm Answer Key

UCSC Econ 204A

February 17, 2014

Problem 1

Given the two-player normal form game with payoff bimatrix:

	a	b	c
A	$\mathbf{3}, \mathbf{4}$	$4, 2$	$\mathbf{7}, 0$
B	$1, 5$	$\mathbf{5}, \mathbf{6}$	$3, 1$
C	$2, 1$	$3, \mathbf{3}$	$6, 2$

(a) **Does either player have a dominant strategy?**
No. By comparing each players' actions, we can see that no strategy dominates the other two for either player 1 or 2.

(b) **Does either player have a dominated strategy?**
Yes. By examining payoffs for actions, we can see that c is dominated for player 2 by b, and, because we eliminate c, we may also say that C is dominated for player 1 by action A. This is the IDSDS method.

(c) **Find all Nash equilibria in pure strategies, if any.**
All best responses are in bold. Recall that we have discovered some dominated strategies which gives a reduced matrix:

	a	b
A	$\mathbf{3}, \mathbf{4}$	$4, 2$
B	$1, 5$	$\mathbf{5}, \mathbf{6}$

Any cell in which both payoffs are bold is a pure strategy NE. Thus (A, a) and (B, b) are pure NE.

(d) **Find all Nash equilibria in mixed strategies, if any.**
We suspect there is a mixed NE here because there is an even number of pure NE. To determine the mix, assign a probability of p to the strategy A and $1 - p$ to B. Similarly, assign q to a and $1 - q$ to b. We calculate:

$$3q + 4(1-q) = 1q + 5(1-q)$$
$$\Rightarrow q = \frac{1}{3}$$

$$4p + 5(1-p) = 2p + 6(1-p)$$
$$\Rightarrow p = \frac{1}{3}$$

Hence our mixed strategy NE is $\left(\frac{1}{3}A + \frac{2}{3}B, \frac{1}{3}a + \frac{2}{3}b\right)$

Problem 2

Compute the coefficient of absolute risk aversion (CARA) and coefficient of relative risk aversion (CRRA) for an individual with Bernoulli function $u(x) = ln(10 + x)$

Remember that CARA $= -\dfrac{u''(x)}{u'(x)}$ and CRRA $= -x\dfrac{u''(x)}{u'(x)}$.

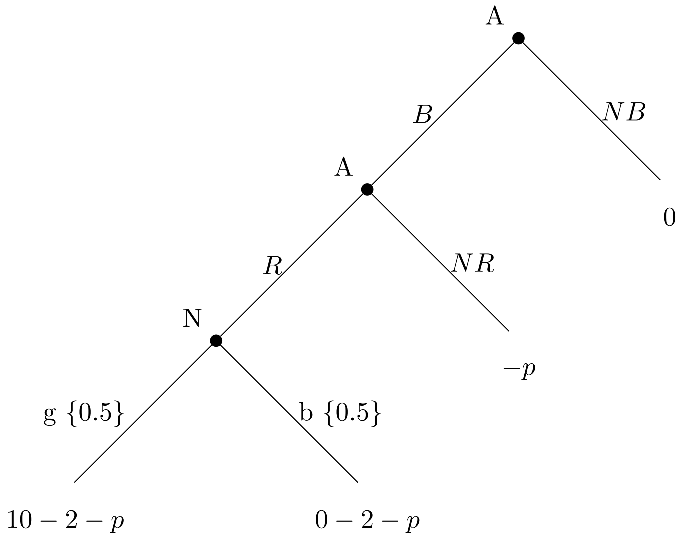
It is easy to compute that $u'(x) = \dfrac{1}{10+x}$ and $u''(x) = -\dfrac{1}{(10+x)^2}$.

$$\text{Then CARA}= \; -\frac{-\frac{1}{(10+x)^2}}{\frac{1}{10+x}} = \frac{1}{10+x}$$

$$\text{Then CRRA}= \; -x\frac{-\frac{1}{(10+x)^2}}{\frac{1}{10+x}} = \frac{x}{10+x}$$

Problem 3

(a) A decision tree for this question is as follows. Let A denote our agent, N denote a nature move, B and NB; buy or not buy the mine, R and NR; produce or not produce, g and b; good and bad revenue, and lastly let p denote the price of the zinc mine.



We may then solve this problem by backward induction. Note that risk neutral agent cares only about expected values.

First, since the last node is nature's move, we calculate the expected value of this event:

$$\frac{1}{2}(10-2-p)+\frac{1}{2}(0-2-p)=3-p$$

At the second node, the agent compares $(3-p)$ to $(-p)$, and chooses $\{R\}$; to produce.

At the first node, the agent compares $(3-p)$ to 0. In this case **the agent will choose {B}; to buy, if $p \leq 3$**.

For the future use, the expected payoff of the agent is $\max\{3-p, 0\}$.

(b) Now, since the agent is not risk neutral in this case, he cares about expected utility. So, we replace payoff x by $u(x)=\ln(10+x)$ in the decision tree.

At the last node, the expected utility is:

$$\frac{1}{2}\ln(10+10-2-p)+\frac{1}{2}\ln(10+0-2-p)=\frac{1}{2}\ln(18-p)+\frac{1}{2}\ln(8-p)$$

At the second node, the agent compares $\frac{1}{2}\ln(18-p)+\frac{1}{2}\ln(8-p)$ to $\ln(10-p)$, and **chooses**

$$\begin{cases} R & \text{if } p \leq \frac{22}{3} \\ NR & \text{otherwise} \end{cases}$$

Where $\frac{22}{3}$ is calculated as follows:

$$\frac{1}{2}\ln(18-p)+\frac{1}{2}\ln(8-p)=\ln(10-p)$$

$$\Rightarrow e^{18-p}e^{8-p}=e^{(10-p)^2}$$

$$\Rightarrow 44-6p=0$$

$$\Rightarrow p=\frac{22}{3}$$

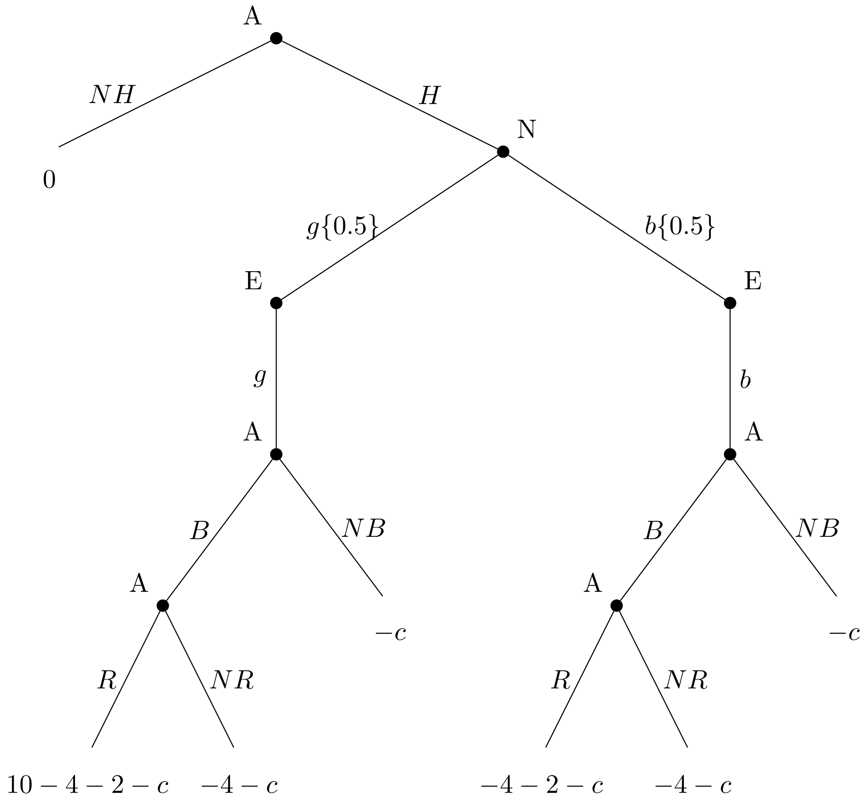
We thus focus on the nontrivial case where $p \leq \frac{22}{3}$. Then at the first node, the agent compares $\frac{1}{2}\ln(18-p)+\frac{1}{2}\ln(8-p)$ to $\ln(10)$, and **chooses B if**

$$p \leq 13-5\sqrt{5} \approx 1.82$$

By definition, **risk premium is equal to:**

$$\ln 13 - \frac{1}{2}(\ln 18 + \ln 8)$$

(c) First, since p= \$4 here, the expected payoff without hiring the expert is 0 from part (a).
Then, the decision tree for this problem is as follows where H and NH represent Hire or not hire the expert (E) and c is the cost of hiring.



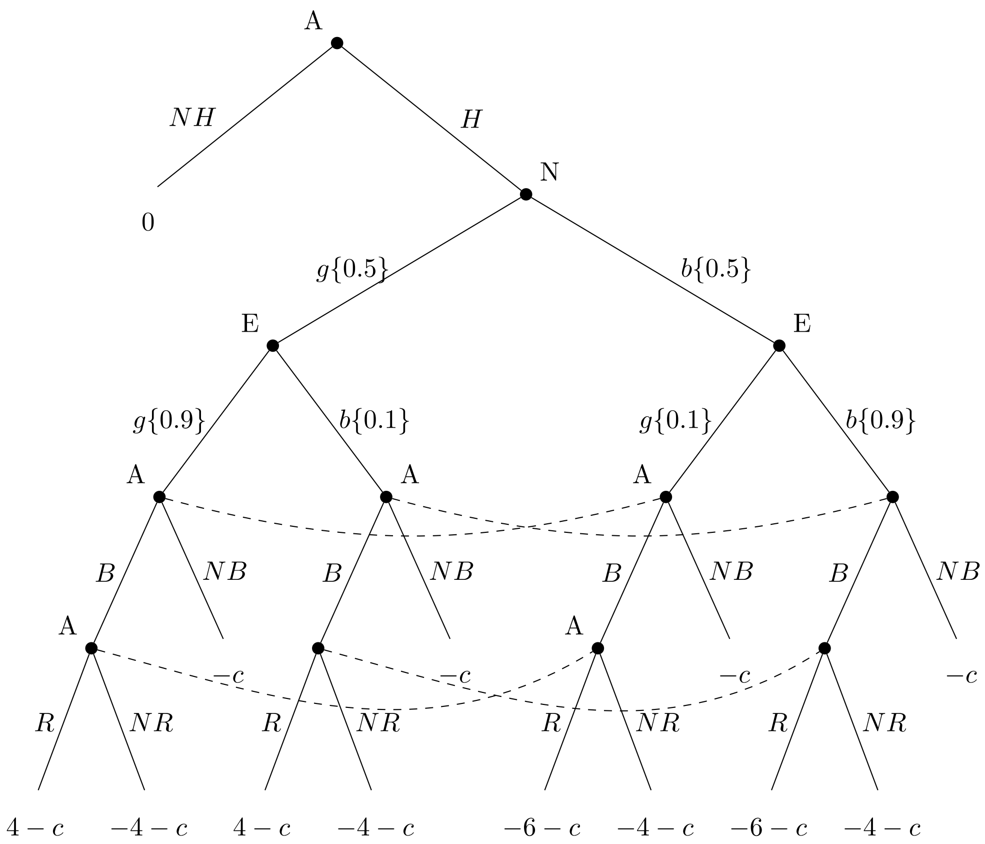
We can easily confirm that if the expert forecasts good, the agent chooses R and then B (produce and buy) and payoff is $10 - 4 - 2 - c$ and if the expert forecasts bad, then the agent chooses to not buy, NB and his payoff is $-c$.

At the second node on the right, expected payoff is

$$\frac{1}{2}(10 - 4 - 2 - c) + \frac{1}{2}(-c) = 2 - c$$

Thus, **the agent chooses to hire, H if $c \leq 2$.**

(d) Now, the forecasts from expert is not perfect. The decision tree is as follows.



For the agent, there are two possible messages; good or bad. First, we have to verify the agent's optimal decision conditional on the forecast.

If the forecast is good, we can immediately see $\{B, NR\}$ won't be chosen. **Then, the agent chooses {B, R} if:**

$$P[g_s|g_f](4-c) + P[b_s|g_f](-6-c) \geq -c$$

where g_f represents a forecast of "good" and b_s represents the state of "bad", etc.

By Bayes' theorem

$$\begin{aligned} P[g_s|g_f] &= \frac{P[g_f|g_s]P[g_s]}{P[g_s]} \\ &= p[g_f|g_s] = 0.9 \end{aligned}$$

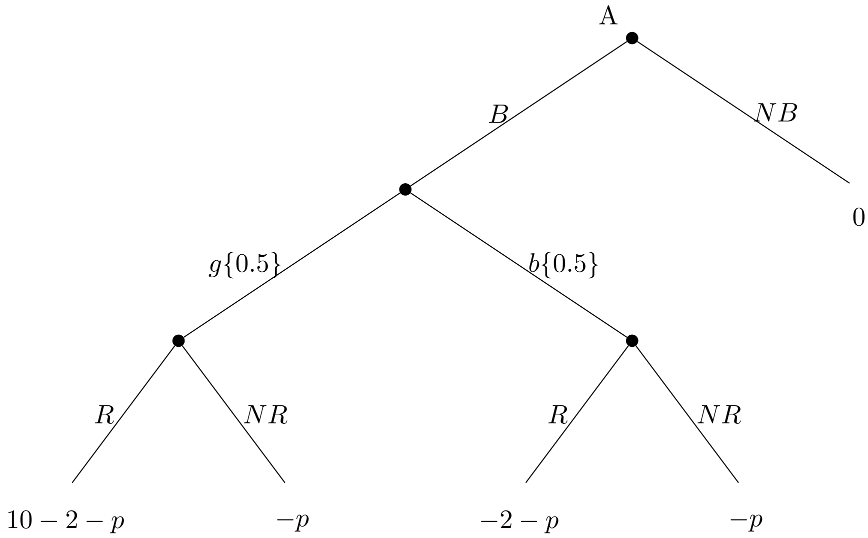
As a result, **the agent chooses {B, R} given good forecast** (g_f).

In the same way, we can verify that **the agent chooses {NB} given bad forecast** (b_f). Thus, the expected payoff from {H} is

$$\frac{1}{2}(0.9(4-c) + 0.1(-6-c)) + \frac{1}{2}(0.9(-c) + 0.1(-c)) = 1.5 - c$$

So, **the agent chooses {Hire} if $c \leq 1.5$**.

(e) Now we can defer the production cost, p, until after determining the revenue. The decision tree is as follows.



We can immediately see that **the agent chooses $\{R\}$ if the state is good and chooses $\{NR\}$ if the state is bad.** The expected payoff from $\{B\}$ is

$$\frac{1}{2}(10-2-p)+\frac{1}{2}(-p)=4-p$$

Therefore, by comparing (4-p) and 0, **the agent chooses$\{B\}$ if $p \leq 4$**.

(f) (extra credit)
Now we just have to calculate expected utility instead of expected payoff in part (c).
The agent chooses {H} if

$$\frac{1}{2}\ln(10+4-c)+\frac{1}{2}(10-c)\geq \ln 10$$
$$\Leftrightarrow c \leq 12-2\sqrt{26}\approx 1.8$$

We can see that WTP is smaller than part (c).

Problem 4

Part (a)

Denote Professor as player 1, and student as player 2. For EFG, see Figure 1.

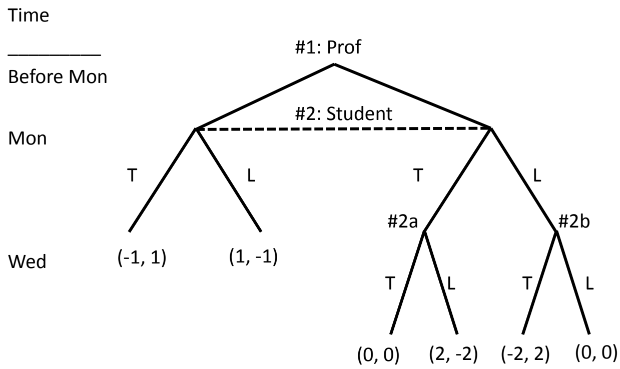


Figure 1: Extensive Form Game

Part (b)

$S_1 = \{M, W\}$
$S_2 = \{TTT, TTL, TLT, TLL, LTT, LTL, LLT, LLL\}$
where $\{ABC\}$ means student plays A on Monday, then (if the game does not end), plays B if he played T on Monday, or plays C if he played L on Monday.
The NFG is

	TTT	TTL	TLT	TLL	LTT	LTL	LLT	LLL
M	(-1,$\underline{1}$)	(-1,$\underline{1}$)	(-1,$\underline{1}$)	(-1,$\underline{1}$)	($\underline{1}$,-1)	($\underline{1}$,-1)	($\underline{1}$,-1)	($\underline{1}$,-1)
W	($\underline{0}$,0)	($\underline{0}$,0)	($\underline{2}$, -2)	($\underline{2}$, -2)	(-2,$\underline{2}$)	(0,$\underline{0}$)	(-2,$\underline{2}$)	(0,$\underline{0}$)

Part (c)

As is shown above, the best response is marked. So, there is no pure NE.

Part (d)

Note that student's strategies $\{TLT, TLL, LTT, LLT\}$ are dominated so those four may be eliminated and the remaining four strategies may be simplified to two remaining strategies. Intuitively, once the student observes that there was no test on Monday, he realizes that there must be a test on Wednesday. Consequently, in Wednesday's game, the student will definitely choose T. Thus, the only choice that the student needs to make is to choose T or L on Monday (and if after Monday the game continues so choose T.).

Therefore, a simplified game is: $S_1 = \{M, W\}$, $S_2 = \{T, L\}$ and the NFG is

	T	L
M	(-1,1)	(1,-1)
W	(0,0)	(-2,2)

Denote p as the probability of Professor choosing M, and denote q as the probability of student choosing T. Then, p makes the student indifferent between choosing T and L, and q makes the Professor indifferent between M and W.

$$1 \times p + 0 \times (1 - p) = (-1) \times p + 2 \times (1 - p) \qquad \rightarrow \qquad p = 1/2$$

$$(-1) \times q + 1 \times (1 - q) = 0 \times q + (-2) \times (1 - q) \qquad \rightarrow \qquad q = 3/4$$

Therefore, mixed NE is $(\frac{1}{2}M + \frac{1}{2}W, \frac{3}{4}T + \frac{1}{4}L)$