

### Midterm Exam

**Instructions:** In class, 95 minutes, closed book except for official cheat sheet. Write your name on every page you turn in. Partial credit will be granted for brief, relevant remarks and for partial results -- if you get tangled in algebra somewhere, at least tell me what you know. **When information is insufficient**, please write down a reasonable assumption and proceed. Points as marked; total is 50.

1. You have the opportunity to invest 1 (\$ million) in a startup company, which will allow it to release a beta version of its product. After seeing whether the beta version is good ( $z=a$ ) or bad ( $z=b$ ), you can decide whether to invest another 1 (\$ million). With overall probability  $p(A) = .25$  the company will then succeed and your gross payoff will be 7; otherwise ( $p(B) = .75$ ) it will be worthless. The likelihoods of a good beta version for a product ultimately fated for success and for failure are  $p(a|A) = 0.9$  and  $p(a|B) = 0.3$ .
  - a. Compute the posterior probabilities of success (A) and failure (B) following a good (a) or bad (b) beta version. Also compute the message probabilities  $p(a)$  and  $p(b)$ . (3 pts)
  - b. Draw your decision tree and solve it. Write out your optimal plan. (3 pts)
  - c. Now suppose that a beta version will not be available after all, and you have to decide now whether or not to invest 2; the overall probabilities and gross payoffs are unaffected. Draw and solve the revised decision tree. (2 pts)
  - d. What is the information value of the beta version? (1 pt)
2. Consider the lottery L that pays 100 with probability 0.4, and pays 0 otherwise.
  - a. What is the mean and variance of this lottery? (2 pts)
  - b. Write down a lottery with the same possible payoffs that first-order stochastically dominates L, and another lottery (with two possible payoffs, not necessarily the same as in L) that second-order stochastically dominates L. (2 pts)
  - c. What is the most that a person with Bernoulli function  $u(x) = \sqrt{x}$  would pay to play lottery L? What is her risk premium? (3 pts)
  - d. Compute this person's coefficients of absolute and of relative risk aversion at the mean of L. Which coefficient (if either) has the same value at  $x=100$ ? (3 pts)

3. Consider the two player game described by the following payoff bimatrix.

	a	b	c
A	4,6	1,7	3,8
B	5,2	2,3	4,1
C	6,4	0,4	6,5

- a. Does either player have a dominant strategy? (1 pt)
- b. Does either player have a dominated strategy? (1 pt)
- c. Which strategy profiles survive iterated deletion of strictly dominated strategies? Explain each deletion very briefly. (2 points)
- d. Find all Nash equilibria (NE) in pure strategies, if any. (2 pts)
- e. Find all Nash equilibria (NE) in mixed strategies, if any. (2 pts)
- f. Find all payoff dominant NE and all risk dominant NE, if any. (2 pts)

4. Adrian is thinking of asking Bailey out on a date. Adrian thinks that there are three possibilities for Bailey: i) with probability  $p > 0$  Bailey is nice and likes Adrian, ii) with probability  $q > 0$  Bailey is nice, but doesn't like Adrian, and iii) with probability  $1 - p - q > 0$  Bailey is mean and does not like Adrian. (Adrian believes that a mean Bailey would like him or her with probability zero.) If Adrian doesn't ask Bailey on a date the payoff is 0 for both. If Adrian does ask Bailey then Bailey can accept or reject. If Bailey is nice and likes Adrian, the payoff vector if Bailey accepts is (5,4) where 5 is Adrian's payoff and 4 is Bailey's, while a rejection in this case gives a payoff vector of (-1,0). If Bailey is nice and doesn't like Adrian, the payoff vector from a Bailey acceptance is (4,-1), and is (-1, 0) from a Bailey rejection. If Bailey is mean and doesn't like Adrian, the payoff vector from a Bailey acceptance is (-1,-1) and the payoff vector from a Bailey rejection is (-10,2), because a mean Bailey might advertise the rejection around school and hurt Adrian's chances of getting a date with anyone else.
- Draw the EFG for this strategic situation. (3 pts)
  - Write down the corresponding NFG. (3 pts)
  - Find all pure strategy NE of the NFG. (2 pts)
  - Find all subgames of the EFG. (2 pts)
  - Find a (Bayesian) NE that is subgame perfect when  $p = 1/3$  and  $q = 1/3$ . (3 pts)
  - Find the necessary conditions on  $p$  and  $q$  for there to be a Bayesian Nash Equilibrium in which Adrian asks Bailey on a date. (2 pts)
5. a. Sketch a game form to represent the following situation: Player B chooses whether or not to tell Player A a joke that is unkind to Republicans. B is uncertain whether or not player A is a Republican. (3 pts)
- b. Now sketch a game form in which, after hearing the joke, A is uncertain whether or not B realizes that A indeed is a Republican. (3 pts)