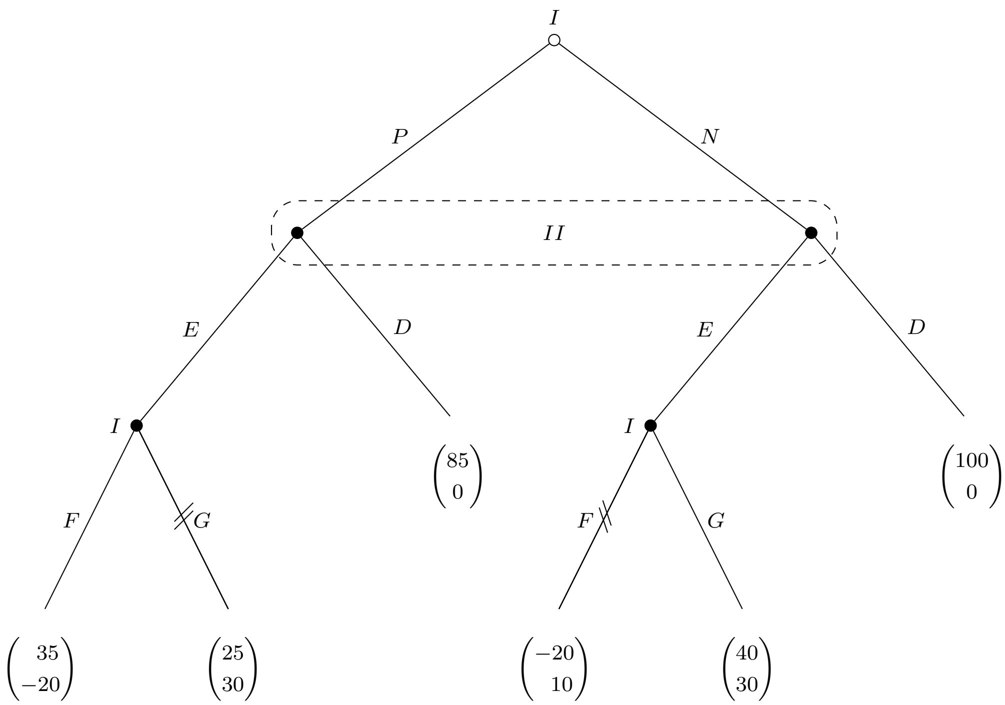
Problem 1

(a)



Player I is incumbent and Player II is potential entrant.

(b)

$$\begin{aligned} E \in BR_{II} &\Leftrightarrow p(-20) + (1-p)30 \geq 0 \\ &\Leftrightarrow 30 \geq 50p \\ &\Leftrightarrow p \leq \frac{3}{5} \end{aligned}$$

That is, Player II (potential entrant) will choose to enter if $p \leq \frac{3}{5}$.

(c)

The first step of backward induction (BI) is shown in the game tree in part (a). The remaining normal form game (NFG) is

		II: E	II: D
I	PF_PG_N	$35, -20$	$85, \underline{0}$
	NF_PG_N	$\underline{40}, \underline{30}$	$\underline{100}, 0$

So the SPNE is (NF_PG_N, E) with $p = 0$.

That is, for the incumbent, it is a (weakly) dominant strategy and subgame perfect (SGP) to not prepare (N), to fight if prepared (F_P, not on the equilibrium path) and to go easy if not prepared (G_N) and the entrant's best response is to enter (E).

Problem 2

(a)

Simple BI gives $(Out_{P_L C_L}, Out_{P_H C_L}, In_{P_H C_H}, In_{P_L C_H})$ for entrant, thus $(P_H|C_H, P_L|C_L)$ for incumbent, with expected payoffs $(1,1)_{C_H} \cdot (.2) + (3,1)_{C_L} \cdot (.8) = (2.6, 1)$.

(b)

1) Try $(P_H|C_H, P_L|C_L)$.

   So the beliefs can be updated as $\mu(C_H|P_H) = 1$ and $\mu(C_L|P_L)$.

   Then $\{BR_2(P_H) = In, BR_2(P_L) = Out\}^{\cdots(*)}$, but BR_1 to $(*)$ includes $P_L|C_H$, breaking this equilibrium.

2) Try $(P_L|C_H, P_H|C_L)$.

   So the beliefs can be updated as $\mu(C_H|P_L) = 1$ and $\mu(C_L|P_H)$.

   Then $\{BR_2(P_H) = Out, BR_2(P_L) = In\}^{\cdots(**)}$, but BR_1 to $(**)$ includes $P_H|C_H$, breaking this equilibrium.

Thus, neither possible pooling strategy is part of a PBE.

(c)

1) Try $(P_H|C_H, C_L)$.

   So the beliefs are $\mu(C_H|P_H) = .2$ (the prior) and $\mu(C_H|P_L) = q \in [0,1]$ (i.e. arbitrary).

   Then, $BR_2(P_H) = Out$ and $BR_2(P_L) = In$ iff $q \geq .5$.

   Then, $BR_1(C_H) = P_H$ and $BR_1(C_L) = P_H$ if $q \geq .5$.

   So a pooling PBE is

$$\left\{m^* = (P_H|C_H, C_L); \mu(\cdot|P_H) = prior, \mu(C_H|P_L) = q \geq 0.5; a^*(P_L) = In, a^*(P_H) = Out\right\}.$$

2) Try $(P_L|C_H, C_L)$.

   So the beliefs are $\mu(C_H|P_L) = prior$ and $\mu(C_H|P_H) = q \in [0,1]$ (i.e. arbitrary).

   Then, $BR_2(P_L) = Out$ and $BR_2(P_H) = In$ iff $q \geq .5$.

   Then, $BR_1(C_L) = P_L$ and $BR_1(C_H) = P_L$ if $q \geq .5$.

   So again we have a pooling PBE as

$$\left\{m^* = (P_L|C_H, C_L); \mu(\cdot|P_L) = prior, \mu(C_H|P_H) = q \geq 0.5; a^*(P_L) = Out, a^*(P_H) = In\right\}.$$

Problem 3

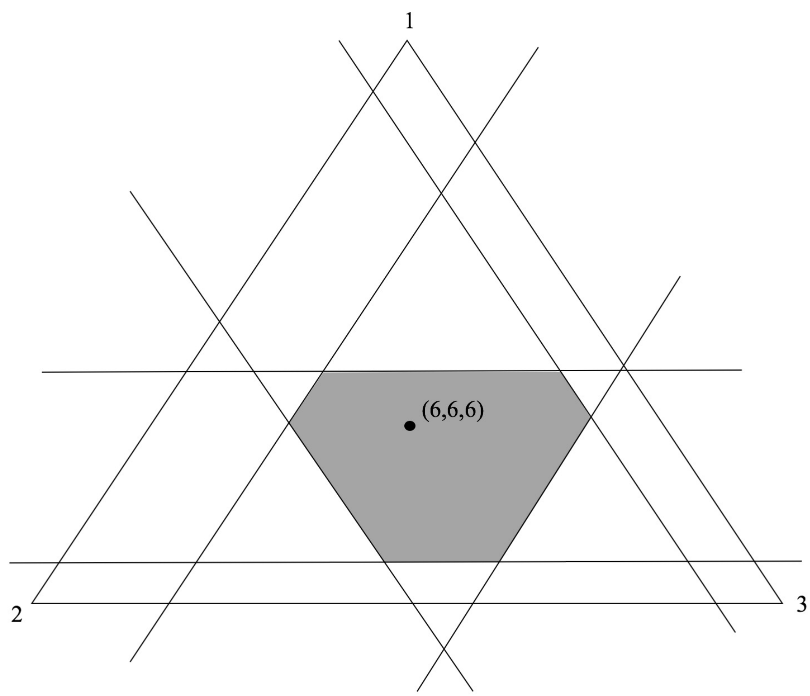
(a)

$$w(\phi) = 0 \qquad w(\{1\}) = 1 \qquad w(\{1,2\}) = 6 \qquad w(\{1,2,3\}) = 18$$
$$w(\{2\}) = 2 \qquad w(\{2,3\}) = 10$$
$$w(\{3\}) = 3 \qquad w(\{1,3\}) = 8$$

The core is

$$x_1 \in [1, 8]$$
$$x_2 \in [2, 10]$$
$$x_3 \in [3, 12].$$

An example is $(6, 6, 6)$.



(b)

ρ	MC_1	MC_2	MC_3
123	1	5	12
132	1	10	7
213	4	2	12
231	8	2	8
312	5	10	3
321	8	7	3
$\sum$	27	36	45
ϕ_i	9/2	6	15/2

The normalized Shapley values are $(1/4, 1/3, 5/12)$.

(c)

Yes, since w is convex (supermodular), $\phi(w) \in Core(w)$.

(d)

The NBS solves

$$\begin{aligned} &\max_{x_1,x_2,x_3} (x_1-1)(x_2-2)(x_3-3) \quad \text{s.t.} \quad x_1+x_2+x_3=18 \\ \Leftrightarrow &\max_{y_1,y_2,y_3} y_1y_2y_3 \quad \text{s.t.} \quad y_1+y_2+y_3=12 \\ \Rightarrow &\quad y_i = 4 \quad \text{for } i=1,2,3 \\ \Leftrightarrow &\quad x_1=5, x_2=6, x_3=7 \end{aligned}$$

Problem 4

(a)

$$w_i = CE_i = \mu_i + 0.2\sigma_i^2 = \begin{cases} 1+0.2\cdot 1^2 = 1.2 & (i=L) \\ 2+0.2\cdot 2^2 = 2.8 & (i=H) \end{cases}$$

(b)

$$E(loss) = (.4)2 + (.6)1 = 1.4 = P$$

(c)

At $P = 1.4$, low risk people refuse $(1.2 < 1.4)$, so only H-type people accept. Then,

$$E(profit) = 4000(P - E(loss|H) = 4000(1.4-2) = -2400,$$

which is \$ 2.4 million loss.

(d)

Assuming a uniform price, insurers will serve only H types (as just seen) at the price $P = 2 + .4 = 2.4$.

(e)

With free entry, P gets down to zero-profit level, so $P = 2$.

(f)

Use screening model and find the insurance company's participation constraint (PC) to separate contracts aimed at H-types and L-types.THe PC's imply that an upper bound in profit for each H-type customer is $(0.2)2^2 = 0.8$ and for each L-type customer is $(0.2)1^2 = 0.2$, or $(0.2)6,000 + (0.8)4,000 = 44,000$, which is \$ 44 million.

(g)

U is not equivalent to Eu, as explained in the Notes 1 (p.24+). It is equivalent up to second order. OVer a limited rantge, the function $u(x) = x - cx^2$ works. See also Problem 2 of Problem Set 1.

Problem 5

(a)

Yes, it is symmetric in the column player's payoff matrix is the transpose of the row player's.

(b)

For $x \in (-2, 0)$, we have $p^* = \frac{a_2}{a_1+a_2} = \frac{x}{-2+x} \in (0, 1)$, e.g., $p^* = 1/3$ for $x = -1$. It is downcrossing since $0 > a_1 = 3 - 5$ and $0 > a_2$, hence a unique, stable NE.

(c)

For $x \in (0, 10)$, $a_2 = x > 0 > a_1 = -2$, hence s_2 is a dominant strategy. Therefore, the pure NE s_2 is globally stable.

(d)

Since $a_1 = -2 < 0$, the CO case with two pure NE is not possible.

(e)

With $x = 1$, (s_1, s_2) is the stage game NE. To sustain cooperation, consider grim trigger strategy: play s_1 untill someone first plays s_2, then play s_2 ever after.
Playing s_1 (or trigger) against trigger yields stream $3, 3, 3, ...$ $\quad (*)$.
Playing s_2 against trigger yields stream $5, 1, 1, ...$ $\quad (**)$.

$(*)$ is BR ($\therefore$ $(trigger, trigger) \in NE$) iff

$$\begin{aligned} PV(*) \geq PV(**) &\Leftrightarrow \frac{3}{1-\delta} \geq 5 + \frac{1}{1-\delta} \\ &\Leftrightarrow 2 \geq 5(1-\delta) \\ &\Leftrightarrow \delta \geq \frac{3}{5}. \end{aligned}$$

If $\delta = \frac{q}{1+r}$, the the condition is $q \geq \frac{3}{5}(1+r)$.