Economics 204 B
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UCSC

Final Exam

1. In A.C. Doyle's famous story "Silver Blaze," detective Sherlock Holmes solves the case by pointing out that the guard dog didn't bark at the horse thief. In 204b terminology, there are two states of Nature -- {Dog knows Thief, Dog doesn't know Thief} --, and two possible actions by the Dog -- {Bark, Don't Bark}.
   a. VERY briefly, define the 204b terms: adverse selection, signaling and screening (6 pts)

(a) adverse selection is the (unfortunate) event which is a common by-product of an asymetri game where the uninformed player would like a particular result (say hard worker or low-risk client) but because of uninformedness, gets the lowtype or high-risk client instead ✓

signaling is a way for an informed player in an asymetric game to let an uninformed player know his type for the purpose of differentiation from other types.

Screening is a way for an uninformed player in an asym game to differentiate types of the informed player ✓

   b. Can any of these terms help explain the inference Sherlock made from the "curious instance" of the dog not barking? In your answer, please refer to pooling versus separating equilibrium, even if you can't relate either of them to the non-barking dog. (8 pts)

(b) Yes... In this case we can use the idea of signaling where the dog is the informed & Sherlock is uninformed. A thief would like a pooling equilibrium (dog barks at everyone or no one) and sherlock determined, however, that this was a seperating equilibrium in which the dog signalled a person was a thief he knew by "not Barking" and would have "Barked" if it was a thief he didn't know.

2. Two firms, Ace and Best, produce brisquets at respective marginal costs $c_A = 4$ and $c_B = 6$. Consumers treat the two brands of brisquets as perfect substitutes.

a. Suppose that the two firms independently choose output and that inverse demand is $p = 56 - 2Q$, where $Q = q_A + q_B$ is the total output for the two firms. Write down the payoff (i.e., profit) functions for the two firms, find their best responses and the Nash equilibrium outputs and profits. Be sure to mention any other assumptions required to obtain your answer. (10pts)

$C_A = 4$
$C_B = 6$

(a) choose q_A, q_B (cournot)

$P = 56 - 2q_A - 2q_B$

$\boxed{\Pi_A = (P - C_A)q_A = (56 - 2q_A - 2q_B - C_A)q_A}$

FOC wrt q_A: $56 - 4q_A - 2q_B - C_A = 0$

$\Rightarrow 4q_A = 56 - 2q_B - C_A$

$\Rightarrow q_A = \dfrac{56 - 2q_B - C_A}{4}$ $\left(= \dfrac{52 - 2q_B}{4} = 13 - \frac{1}{2}q_B \right)$

$\boxed{\Pi_B = (P - C_B)q_B = (56 - 2q_B - 2q_A - C_B)q_B}$

FOC wrt q_B: $56 - 4q_B - 2q_A - C_B = 0$

$\Rightarrow 4q_B = 56 - 2q_A - C_B$

$\Rightarrow q_B = \dfrac{56 - 2q_A - C_B}{4}$ $\left\{ = \dfrac{50 - 2q_A}{4} = 12.5 - \frac{1}{2}q_A \right\}$

SO $\boxed{BR_A(q_B) = 13 - \frac{1}{2}q_B}$

$\boxed{BR_B(q_A) = 12.5 - \frac{1}{2}q_A}$

solve for q_A^*, q_B^*:

$q_A^* = 13 - \frac{1}{2}q_B^*$

$q_B^* = 12.5 - \frac{1}{2}q_A^*$

$q_B^* = 12.5 - \frac{1}{2}(13 - \frac{1}{2}q_B^*) = 12.5 - 6.5 + \frac{1}{4}q_B^*$

$\Rightarrow \frac{3}{4}q_B^* = 6 \Rightarrow \boxed{q_B^* = 8}$ ✓

$q_A^* = 13 - \frac{1}{2}(8)$
$= 13 - 4$
$\boxed{q_A^* = 9}$ ✓

Then $P = 56 - 2(9) - 2(8) = 56 - 18 - 16$
$= 22$

SO $\Pi_A = (22 - 4)(9)$
$= 18 \cdot 9$
$= \boxed{\$162}$ ✓

$\Pi_B = (22 - 6)(8)$
$= 16 \cdot 8$
$= \boxed{\$128}$ ✓

Assuming NO fixed costs! ✓

b. Now suppose that the two firms independently choose price, and the lowest price firm gains the entire market. Assume that this firm sells a quantity consistent with the inverse demand function above. Again find the payoff functions for the two firms, and find their best responses and the Nash equilibrium prices and profits. Be sure to mention any other assumptions required to obtain your answer. (8pts)

b) choose P_A, P_B (Bertrand)

$$\Pi_A = \begin{cases} (P_A - C_A)\left(\frac{56-P_A}{2}\right) & P_A < P_B \\ \frac{1}{2}(P_A - C_A)\left(\frac{56-P_A}{2}\right) & P_A = P_B \\ 0 & P_A > P_B \end{cases}$$

$$\Pi_B = \begin{cases} (P_B - C_B)\left(\frac{56-P_B}{2}\right) & P_B < P_A \\ \frac{1}{2}(P_B - C_B)\left(\frac{56-P_B}{2}\right) & P_B = P_A \\ 0 & P_B > P_A \end{cases}$$

The Best response for each player can't be found from focs (b/c discontinuous), but we know that each player i will try to play $P_i = P_j - \varepsilon$; $\varepsilon > 0$ (arbitrarily small). In this way
if below monopoly price
i would gain the whole market. This continues until $P_i = C_i$.

In this case since $C_A \neq C_B$, "Best" will not produce below $P_B = 6 = C_B$ (because $\Pi_B = 0$ is preferable to $\Pi_B < 0$). Since "Ace's" $MC = 4 < 6$, he is willing to price $P_A < 6$. Thus in → equilibrium, we have that:

$$\begin{cases} P_A = 6 - \varepsilon \approx 6 \\ P_B = 6 \end{cases}$$

And A takes the whole market whilst B gets none. Profits are:

$$Q = \frac{56 - P_A}{2} = \frac{56-6}{2} = 25$$

$$\Pi_A = (P_A - C_A)(25) = (6-4)(25) = \boxed{\$50}$$

$$\Pi_B = (P_B - C_B)(0) = \boxed{\$0}$$

Again, assuming no fixed costs 8/8

which sort of industries is the game in part a more descriptive than the game rt b? Explain very briefly. (2pts)

I guess this would be a better model if a company had capacity concerns/constraints like maybe be a factory where you can't choose price b/c you ~~may not be able to produce that quantity.~~

3. An industry consists of a large population of firms, each of which must choose one of two alternative technologies. The first technology has decreasing returns to scale when rare and increasing returns when common; its profitability can be expressed as $u_1 = 2s_1^2 - 2s_1 + 1$, where s_1 is the fraction of industry output produced using that technology. Technology 2 has moderately decreasing returns at all scales; its profitability is $u_2 = 0.5(1.5 - s_2) = 0.5(s_1 + 0.5)$ when fraction $s_2 = 1 - s_1$ of the output is produced using it.

a. Write down the payoff difference $D = u_1 - u_2$ as a function of s_1, and graph this function. (6pts)

TWO technologies

tech 1 $u_1 = 2s_1^2 - 2s_1 + 1$

tech 2 $u_2 = 0.5(s_1 + 0.5)$

when fraction s_1 of output is produced using it.

(a) $D = u_1 - u_2$

$= 2s_1^2 - 2s_1 + 1 - \left[\frac{1}{2}\left(s_1 + \frac{1}{2}\right)\right]$

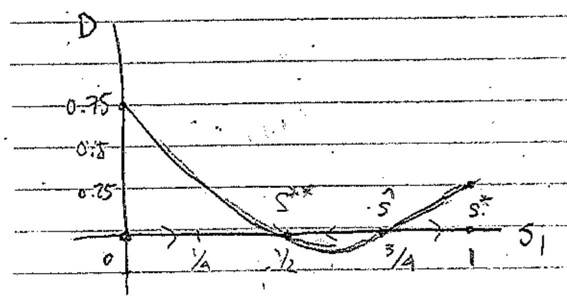
$= 2s_1^2 - 2s_1 + 1 - \frac{1}{2}s_1 - \frac{1}{4}$

$\boxed{D = 2s_1^2 - 2.5s_1 + 0.75}$

$2 - 2.5 + 0.75$

$s_1 = \frac{2.5 \pm \sqrt{6.25 - 6}}{2(2)}$

$s_1 = \frac{2.5 \pm 0.5}{4} = \left(\frac{1}{2}, \frac{3}{4}\right)$



6/6

$4s_1 - 2.5 = 0$

$s_1 = \frac{2.5}{4} = 0.625 \Rightarrow D = 2(0.625)^2 - 2.5(0.625) + 0.75 = -0.03125$

b. Does this game have any pure strategy NE? I.e., is $s_1 = 0$ or 1 a NE? Please verify your answer. (4pts)

b) If $s_1 = 0$, then $D(0) = 0.75 > 0$, means people tend to choose s_1, so not a NE.

If $s_1 = 1$, then $D(1) = 0.25 > 0$. means people won't deviate to s_2.

Thus, $s_1 = 1$ is a pure NE.

c. Suppose that sign preserving dynamics describe the evolution of technology adoption in the industry. Find the evolutionary equilibria and their basins of attraction, using the graph from part a. (6pts)

d. Suppose that the second technology is more recent. Predict the long-run shares of the two technologies. (2pts)

Diagram shows that $\hat{s}_1 = 3/4$ separates basins of attraction for $s_1^* = 1$ and $s^{**} = \frac{1}{2}$. (the two EE)

If 2nd tech. is more recent, then initial state is $s_1 = 1$. So evolutionary (sign-preserving) dynamics never leave its basin $\Rightarrow s_1 = 1$ in the LR.

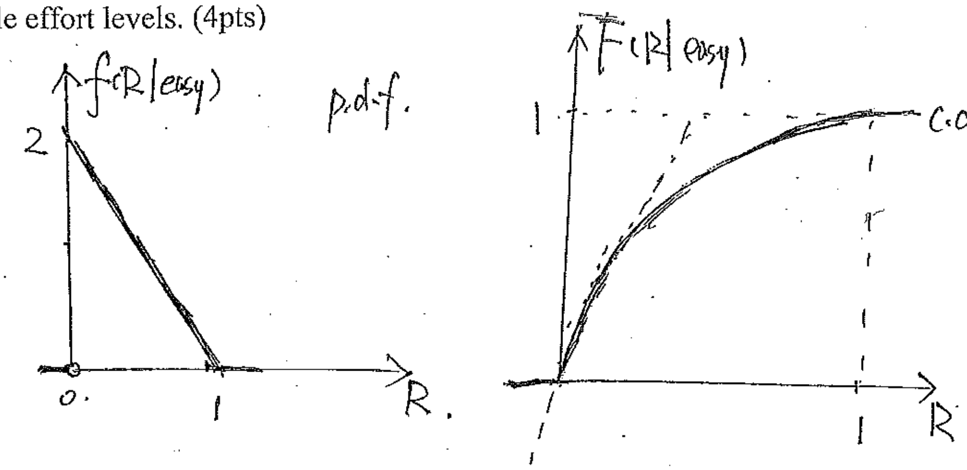
$s_2 = 0$

If a salesman takes it easy, the revenue R he generates in a given month has density f(R) = 2-2R. Alternatively, if he works hard the density is f(R) = 2R. In either case, R lies between 0 and 1 (million dollars).

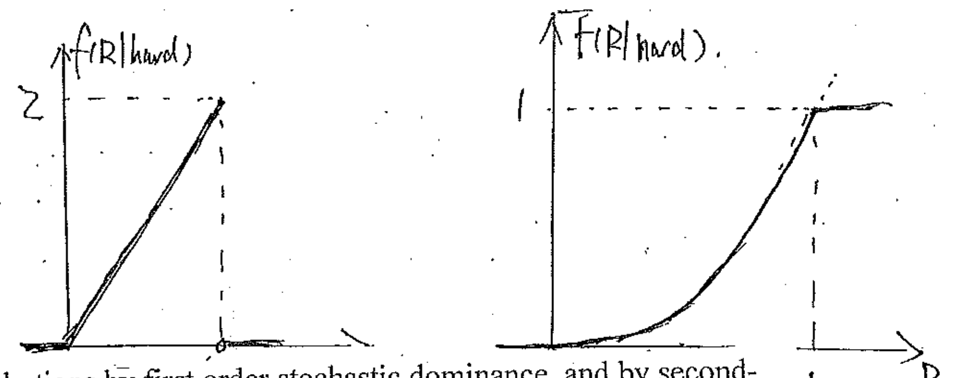
a. Sketch the density and the cumulative distribution function (cdf) for revenue R for the two possible effort levels. (4pts)

a)

Easy:



Hard:



4/4

Rank the two distributions by first order stochastic dominance, and by second-order stochastic dominance. If ranking is not possible in either case, explain why very briefly. (6 pts)

b) • f(·|hard) 1st-order stochastic dominates f(·|easy),

[f(·|hard) is dominant, f(·|easy) is dominated].

because $F(R|easy) \geq F(R|hard)$, $\forall R$. ✓

• f(·|hard) also 2nd-order stochastic dominates f(·|easy). ✗

~~f(·|hard) and f(·|easy) cannot be ranked by 2nd-order stochastic dominance~~

because. $\int_0^s F(s|easy) = \int_0^s -s^2+2s\,ds = -\frac{1}{3}s^3 + s^2.$

$\int_0^s F(s|hard) = \int_0^s x^2 dx = \frac{1}{3}s^3$, $s \in [0,1]$.

SOSD is applied only to distributions w/ same mean! -1

on interval $[0,1]$, $\int_0^s F(s|easy) - F(s|hard)\,ds = -\frac{2}{3}s^3 + s^2 = s^2(1-\frac{2}{3}s) \geq 0.$

on $(-\infty, 0) \cup (1, +\infty)$, $F(\cdot|easy) = F(\cdot|hard)$.

5/6

c. If the salesman has Bernoulli function $v(x) = 2x^{0.5}$ and is paid a 25% commission (i.e., $x = R/4$), then what is his certainty-equivalent for the payment received when working hard? For taking it easy? (6 pts)

• If working hard,

$$E[u(R)] = \int_0^1 f(R|hard)\, u(R)\, dR = \int_0^1 2R \cdot \sqrt{R}\, dR = 2 \cdot \frac{2}{5} R^{5/2} \Big]_0^1 = \frac{4}{5}$$

Let. $u(CE) = 2\sqrt{CE} = \frac{4}{5}$, $CE = \frac{1}{4}\frac{16}{25} = 0.64 \times \frac{1}{4} = 0.16$ (million)

So, the certainty-equivalent for payment is 0.16 (corresponding R is 0.64) million

• If working easy,

$$E[u(R)] = \int_0^1 f(R|easy)\, u(R)\, dR = \int_0^1 (2-2R) \cdot \sqrt{R}\, dR = 2 \cdot \left(\frac{2}{3} R^{3/2} - \frac{2}{5} R^{5/2}\right)\Big]_0^1 = \frac{8}{15}$$

Let $u(CE) = 2\sqrt{CE} = \frac{8}{15}$. $CE = \frac{64}{225} \cdot \frac{1}{4} = \frac{16}{225} \approx 0.071$ million.

So, the certainty-equivalent for payment is 0.071 million (corresponding R is 0.284).

5. The salesman's boss just hired you advise on incentive pay. You believe that the salesman in problem 4 has utility cost g=0 for taking it easy, has utility cost g=0.5 for working hard, and could obtain utility 1.0 if he quit.

a. What is the salesman's optimal effort choice under the current 25% commission plan? Show your work. (4pts)

$$E[\tilde{U}|hard] = E[u|hard] - g_{hard} = \frac{4}{5} - 0.5 = 0.3.$$

If taking it easy.

$$E[\tilde{u}|easy] = E[u|easy] - g_{easy} = \frac{8}{15} - 0 \approx 0.53.$$

If quit,

$$\bar{u} = 1.0$$

So, under current plan, salesman will quit

b. What else (if anything) do you need to know about the salesman or the boss to apply techniques learned in Econ 204b? Be explicit. (4 pts)

· Is e observable?

· Is principal / boss risk neutral?

c. What does the standard formula (involving unknown parameters γ and μ) tell you about how to revise the payment schedule to motivate the salesman to work hard? Be as specific as possible using the unknown parameters. (8 pts)

(c) formula: $\frac{1}{v'(w(\pi))} = \gamma + \mu\left[1 - \frac{f(\pi|e)}{f(\pi|e^*)}\right]$, $e^* \in \{e_L, e_H\}$

Here, Boss wants to motivate hard work so $e^* = e_H$

$v'(w(\pi)) = (v(w))' = (2w^{0.5})' = w^{-0.5}$

$\Rightarrow \frac{1}{w^{-0.5}} = w^{0.5}$ ✓

$\frac{f(R|e_L)}{f(R|e_H)} = \frac{2-2R}{2R} = \frac{1}{R} - 1$ ✓

Let's assume $R \in (0,1]$ s.t. we don't have ∞ problem.
already assumed in Prob 4.

then $1 - \frac{f(R|e_L)}{f(R|e_H)} = 1 - \frac{1}{R} + 1 = 2 - 1/R$

$\Rightarrow w^{0.5} = \gamma + \mu[2 - 1/R]$

d. What are some important caveats to mention to the boss about actually using the formula in c.? (2 pts)

e. For extra credit, time permitting, compute γ and μ and thus the specific payment schedule for the salesman in part c.

(d) In actuality, we may not know things like $v(x)$, $f(\pi|e_L)$, $f(\pi|e_H)$, ... etc. ✓ Even if we did, there could be a negative effect of having a strict wage schedule in the first place. ✓ Maybe the salesman will find a schedule distasteful & might even demand higher compensation if he's forced

6. The characteristic function in a 3 player game gives total payoff 1 to the coalitions K={1,2}, {1,3} and {1,2,3}, and total payoff 0 to all other coalitions.
   a. Is this game convex? (2pts)

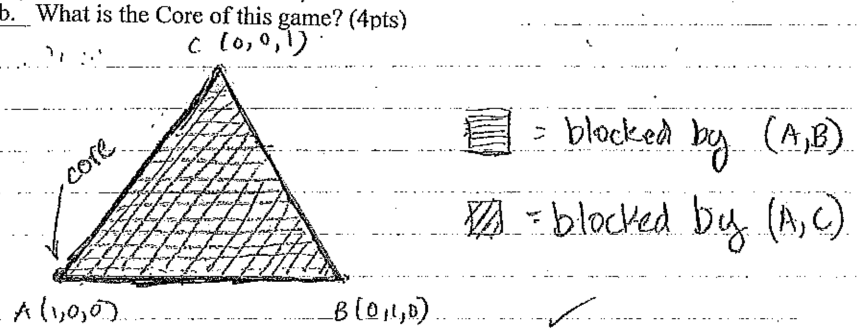
I'm Changing "1,2,3" to "A,B,C" for my own ease ... hope that's okay.

$v\{A\} = v(B) = v(C) = 0$
$v(A,B) = 1$ $v(B,C) = 0$
$v(A,C) = 1$
$v(A,B,C) = 1$

(i) adding B to A increases benefit by 1
(ii) adding B to (A,C) increases by 0.
It should increase by [~~more than~~ at least] one in (ii).
Not convex. ✓

   b. What is the Core of this game? (4pts)

(b)



✓

thus the core is the single point (1,0,0)

   c. What is the Shapley Value of this game? (4pts)

(c)

	A	B	C
ABC	0	1	0
ACB	0	0	1
BAC	1	0	0
BCA	1	0	0
CAB	1	0	0
CBA	1	0	0
total / n!	$\frac{4}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\varphi(A,B,C) = (\frac{2}{3}, \frac{1}{6}, \frac{1}{6})$