

Practice Problems for Final Exam

Instructions. In class, closed book, three hours, one double-sided page of notes allowed. When insufficient information is provided, please write down a plausible specific assumption and proceed to the solution. A numerical solution will be considered complete when it is ready to be entered into a calculator, e.g., $(3/17)*20^{0.5}$ gets as much credit as 0.79. Partial credit will be awarded for partial solutions and for brief, relevant remarks, but not for rambling. Points (pts) are marked (plus 4 pts for writing your name on the first page of your answer sheet); total is 100.

1. In A.C. Doyle's famous story "Silver Blaze," detective Sherlock Holmes solves the case by pointing out that the guard dog didn't bark at the horse thief. In 204b terminology, there are two states of Nature -- {Dog knows Thief, Dog doesn't know Thief} --, and two possible actions by the Dog -- {Bark, Don't Bark}.
 - a. VERY briefly, define the 204b terms: adverse selection, signaling and screening. (6 pts)
 - b. Can any of these terms help explain the inference Sherlock made from the "curious instance" of the dog not barking? In your answer, please refer to pooling versus separating equilibrium, even if you can't relate either of them to the non-barking dog. (8 pts)
2. Two firms, Ace and Best, produce brisquets at respective marginal costs $c_A = 4$ and $c_B = 6$. Consumers treat the two brands of brisquets as perfect substitutes.
 - a. Suppose that the two firms independently choose output and that inverse demand is $p = 56 - 2Q$, where $Q = q_A + q_B$ is the total output for the two firms. Write down the payoff (i.e., profit) functions for the two firms, find their best responses and the Nash equilibrium outputs and profits. Be sure to mention any other assumptions required to obtain your answer. (10pts)
 - b. Now suppose that the two firms independently choose price, and the lowest price firm gains the entire market. Assume that this firm sells a quantity consistent with the inverse demand function above. Again find the payoff functions for the two firms, and find their best responses and the Nash equilibrium prices and profits. Be sure to mention any other assumptions required to obtain your answer. (8pts)
 - c. For which sort of industries is the game in part a more descriptive than the game in part b? Explain very briefly. (2pts)
3. An industry consists of a large population of firms, each of which must choose one of two alternative technologies. The first technology has decreasing returns to scale when rare and increasing returns when common; its profitability can be expressed as $u_1 = 2s_1^2 - 2s_1 + 1$, where s_1 is the fraction of industry output produced using that technology. Technology 2 has moderately decreasing returns at all scales; its profitability is $u_2 = 0.5(1.5 - s_2) = 0.5(s_1 + 0.5)$ when fraction $s_2 = 1 - s_1$ of the output is produced using it.
 - a. Write down the payoff difference $D = u_1 - u_2$ as a function of s_1 , and graph this function. (6pts)
 - b. Does this game have any pure strategy NE? I.e., is $s_1 = 0$ or 1 a NE? Please verify your answer. (4pts)

Please turn over

- c. Suppose that sign preserving dynamics describe the evolution of technology adoption in the industry. Find the evolutionary equilibria and their basins of attraction, using the graph from part a. (6pts)
 - d. Suppose that the second technology is more recent. Predict the long-run shares of the two technologies. (2pts)
4. If a salesman takes it easy, the revenue R he generates in a given month has density $f(R) = 2 - 2R$. Alternatively, if he works hard the density is $f(R) = 2R$. In either case, R lies between 0 and 1 (million dollars).
- a. Sketch the density and the cumulative distribution function (cdf) for revenue R for the two possible effort levels. (4pts)
 - b. Rank the two distributions by first order stochastic dominance, and by second-order stochastic dominance. If ranking is not possible in either case, explain why very briefly. (6 pts)
 - c. If the salesman has Bernoulli function $v(x) = 2x^{0.5}$ and is paid a 25% commission (i.e., $x = R/4$), then what is his certainty-equivalent for the payment received when working hard? For taking it easy? (6 pts)
5. The salesman's boss just hired you advise on incentive pay. You believe that the salesman in problem 4 has utility cost $g=0$ for taking it easy, has utility cost $g=0.5$ for working hard, and could obtain utility 1.0 if he quit.
- a. What is the salesman's optimal effort choice under the current 25% commission plan? Show your work. (4pts)
 - b. What else (if anything) do you need to know about the salesman or the boss to apply techniques learned in Econ 204b? Be explicit. (4 pts)
 - c. What does the standard formula (involving unknown parameters γ and μ) tell you about how to revise the payment schedule to motivate the salesman to work hard? Be as specific as possible using the unknown parameters. (8 pts)
 - d. What are some important caveats to mention to the boss about actually using the formula in c.? (2 pts)
 - e. For extra credit, time permitting, compute γ and μ and thus the specific payment schedule for the salesman in part c.
6. The characteristic function in a 3 player game gives total payoff 1 to the coalitions $K = \{1,2\}$, $\{1,3\}$ and $\{1,2,3\}$, and total payoff 0 to all other coalitions.
- a. Is this game convex? (2pts)
 - b. What is the Core of this game? (4pts)
 - c. What is the Shapley Value of this game? (4pts)