1. For the one-output, two-inputs profits maximization problem, using supermodularity, under what *minimal conditions* is

   (a) the optimal output increasing in p?

   (b) $x_1(p, w)$ decreasing in p? Hint: Rewrite the problem changing the variables of choice from x_1, x_2 to x_1, y, i.e.,

   $$\max_{x_1, y} \{py - w_1 x_1 - w_2 h(x_1, y)\}.$$

   The hardest part of the problem is to provide an expression for $\partial^2 h(x_1, y)/\partial x_1 \partial y$ in terms of the production function!

   (c) $x_1(p, w)$ decreasing in w_1? Hint: Rewrite the problem in terms of sequential optimization, i.e.,

   $$\max_{x_1} \{\max_{x_2} [pf(x_1, x_2) - w_1 x_1 - w_2 x_2]\}.$$

2. A consumer has utility $U(x_1, x_2)$, and buys x_1 at price p_1 and an input z, from which she makes x_2 with a production function $x_2 = f(z)$, at price p_2. Her budget is thus $p_1 x_1 + p_2 z = y$. Use supermodularity to find all needed minimal conditions on U and f to guarantee that $\partial z^*/\partial y > 0$.