

ECON 204A: FALL 2016  
PRODUCER THEORY, CONSUMER THEORY, AND DECISION THEORY  
PROBLEM SET 5

1. Consider the following profit function (which is differentiable as well as concave in  $(x, y)$ ):

$$\tilde{\pi}(x, y) = \alpha tx + ty - x^2 - y^2 - xy$$

where  $t \in [0, \infty)$  is the parameter of interest, and  $\alpha \in (-\infty, \infty)$  is a secondary parameter.

- (a) Suppose that  $y$  is fixed. The firm chooses  $x$  to maximize profits. Is the optimal choice of  $x$  increasing or decreasing in  $t$ ? Does  $\alpha$  play any role here?
  - (b) Suppose that  $x$  is fixed. The firm chooses  $y$  to maximize profits. Is the optimal choice of  $y$  increasing or decreasing in  $t$ ? Does  $\alpha$  play any role here?
  - (c) Now suppose the firm chooses both  $x$  and  $y$ , and that  $\alpha \geq 0$ . Solve for the optimal choices of  $x$  and  $y$  as a function of  $t$  when both  $x$  and  $y$  are choice variables. Discuss the monotonicity of  $x$  and  $y$  in  $t$ , as it depends on the value of  $\alpha$ .
  - (d) Interpret your answers in the context of the monotone comparative statics theorem.
2. Consider a monopolist who has three choice variables: output  $q$ , "marketing"  $m$ , and new equipment  $e$ . The cost of producing  $q$  units of output is given by  $c.q/e$ , where  $c$  is a constant. Equipment is purchased on a competitive market at price  $r$ . The cost of  $m$  units of marketing is  $h(m; \theta)$ , where  $\theta$  is a parameter that increases the incremental cost of marketing (i.e.,  $h$  is supermodular). Demand is given by  $P(q; m)$ , so that revenue is  $q.P(q; m)$ . Assume that  $P(q; m)$  is sufficiently differentiable.

- (a) Write down conditions on the function  $P$  which guarantee that marketing increases marginal revenue.

Assume that your conditions from (a) hold through the rest of the problem. Note that the firm's profit function is given as follows:

$$q.P(q; m) - c.q/e - r.e - h(m; \theta)$$

- (b) Suppose that in the short run, equipment is fixed. How do the firm's short run choices of quantity and marketing change with the parameter  $\theta$ ? With the fixed level of equipment?
- (c) How does the firm long run choice of equipment change with  $\theta$ ?
- (d) In response to a 10% increase in  $\theta$ , will the choices of quantity and marketing change by more in the short or in the long run? Sketch a proof of your answer.
- (e) How do the firm's long run profits change with  $r$ ? Does your answer change with the parameter  $\theta$ ? If so, in what direction? Interpret. (Note: You may want to first use the old standard techniques to get an idea, but the final answer must be derived using supermodularity.)