Producer Theory, Consumer Theory, and Decision Theory

Problem Set 3

1. Suppose that a consumer (with a standard utility function) buys clothing for the warmth, w, it provides and that each unit of clothing provides k units of warmth. Thus k is a measure of the quality of the clothes. Assume that the consumer buys housing for the shelter, s, it provides, and that one unit of housing provides one unit of shelter. If H is the number of housing units and C is the number of clothing units consumed, then the utility function for housing and clothing is $U(H, kC)$. Given the prices of housing and clothing p_h and p_c, respectively, and income y:

   (a) Set up the problem and derive the first order conditions; and

   (b) Under what conditions does an improvement in the quality of clothes increase clothing consumption (i.e. $\partial C^*/\partial k \geq 0$)?

2. Let $U(\mathbf{x}) = \sum_{i=1}^{n} u_i(x_i)$, where $\mathbf{x} \in \mathbb{R}^n_+$. If each u_i is concave and strictly increasing, show that no good can be inferior. (Hint: Think!)

3. Suppose that $U(\mathbf{x})$ is a linear homogenous utility function.

   (a) Show that the expenditure function is multiplicatively separable in $\mathbf{p}$ and u and can be written in the form $e(\mathbf{p}, u) = e(\mathbf{p}, 1) u$.

   (b) Show that the marginal utility of the income depends on $\mathbf{p}$ but is independent of u.

4. The substitution matrix of a utility-maximizing consumer's demand system at prices $(8, p)$ is $\begin{pmatrix} a & b \\ 2 & -1/2 \end{pmatrix}$. Find a, b and p.