(1) $\max\limits_{x_1, x_2} \{ U(x_1, f(x_2)) : p_1 x_1 + p_2 x_2 = (1-t) y \}$

(a) $L = U(x_1, f(x_2)) - \lambda [p_1 x_1 + p_2 x_2 - (1-t) y]$

$$\frac{\partial L}{\partial x_1} = U_1 - \lambda^{\#} p_1 = 0$$

$$\frac{\partial L}{\partial x_2} = U_2 f' - \lambda^{\#} p_2 = 0$$

$$\frac{\partial L}{\partial \lambda} = - p_1 x_1^{\#} - p_2 x_2^{\#} + (1-t) y = 0$$

Diff this system with respect to t.

$$U_{11} \frac{\partial x_1^{\#}}{\partial t} + U_{12} f' \frac{\partial x_2^{\#}}{\partial t} - \frac{\partial \lambda^{\#}}{\partial t} p_1 = 0$$

$$U_{21} f' \frac{\partial x_1^{\#}}{\partial t} + \left[U_{22} f' f' + U_2 f'' \right] \frac{\partial x_2^{\#}}{\partial t} - \frac{\partial \lambda^{\#}}{\partial t} p_2 = 0$$

$$- p_1 \frac{\partial x_1^{\#}}{\partial t} - p_2 \frac{\partial x_2^{\#}}{\partial t} = y$$

$$\Longrightarrow \frac{\partial x_2^{\#}}{\partial t} = \frac{\begin{vmatrix} U_{11} & 0 & -p_1 \\ U_{21} f' & 0 & -p_2 \\ -p_1 & y & 0 \end{vmatrix}}{\underbrace{\begin{vmatrix} U_{11} & U_{12} f' & -p_1 \\ U_{21} f' & U_{22} f' f' + U_2 f'' & -p_2 \\ -p_1 & -p_2 & 0 \end{vmatrix}}_{\text{assume} > 0}}$$

then the sign of $\partial x_2^{\#} / \partial t$ depends on.

$$- y \left(- U_{11} p_2 + U_{21} f' p_1 \right) = y p_2 \left(U_{11} - U_{21} f' \frac{p_1}{p_2} \right)$$

$$\Rightarrow y P_{2} \left(U_{11} - U_{21} f' \frac{U_1}{U_2 f'} \right) = y \frac{P_2}{U_2} \left(U_{11} U_2 - U_{21} U_1 \right)$$

Which means good Z_1 is a normal good

(b) $\frac{\partial V}{\partial t}(p_1, p_2, y) = -\lambda^* y < 0$

assumptions

$U(x_1, f(x_2))$ is st. quasiconcave

$f'(x_2) > 0$

NOTE BOOK

(2) If u is homogeneous of degree 0, then

$$e(p,u) = \min_{x \in \mathbb{R}^n_+} \{ px : u(x) = u \}$$

$$= \min_{x \in \mathbb{R}^n_+} \{ u\, p(x/u) : u(x/u) = 1 \}$$

$$= u \min_{z \in \mathbb{R}^n_+} \{ pz : u(z) = 1 \} = u\, e(p,1)$$

Then (By the envelop thm)

$$x_i^h(p,u) = \frac{\partial e}{\partial p_i}(p,u) = u \frac{\partial e}{\partial p_i}(p,1) = u\, x_i^h(p,1)$$

$$\Longrightarrow \frac{\partial x_i^h}{\partial u}(p,u) = x_i^h(p,1) \geq 0 \quad i = 1, \ldots, n$$

NOTE BOOK

(3) We know that

$$\frac{\partial\left(\frac{\partial C}{\partial y}(w,y)\right)}{\partial w_i} < 0 \Longrightarrow \frac{\partial^2 C(w,y)}{\partial y\,\partial w_i} < 0$$

(By young's thm) (By Shephard's lemma)

$$\frac{\partial^2 C(w,y)}{\partial y\,\partial w_i} = \frac{\partial^2 C(w,y)}{\partial w_i\,\partial y} = \frac{\partial\left(\frac{\partial C}{\partial w_i}(w,y)\right)}{\partial y} =$$

$$= \frac{\partial x_i^*(w,y)}{\partial y} < 0$$

Which means the factor of production x_i is inferior

NOTE BOOK